Mini ising Ti e Stepping Errors in Nu eric Mode s of the At osphere nd Oce n

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A str ct

Due to the wide usage of numerical models in Meteorology it is essential reduce model errors to get better predictions. The model errors are due to space and time-di erencing. The model errors are considered separately with the main focus of the paper on time-di erencing schemes. The Asselin-filtered leapfrog scheme, the proposed modified filters and the Adams-Bashforth family of schemes are emplo.049119(60(s-)-0.336

Acknow edg ents

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4.13 Combined leapfrog and forward step.

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of the globe where observations cannot easily be made. Therefore it depends on the area of the atmosphere you are modelling in order to get accurate initial data. How-

Aims

In this dissertation we will only consider time-differencing schemes and will not consider different space-differencing schemes. This is done to isolate the time-stepping errors. The aims are:

- To implement a number of time-differencing schemes in a simple nonlinear

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Ti e D erencing Sche es

In this Chapter we discuss various time-di□erencing schemes. We first consider the

that we are after. The actual solution is known as the physical mode. Depending on the problem being solved the computational mode can cause the numerical solution to grow exponentially. The rate at which this occurs depends on the time step Δ . Towards the end of the numerical integration the numerical solution deviates further away from the actual solution. The reason for this is the integration goes from the $_{n-1}$ time point to the $_{n+1}$ time point and misses out at even and odd time steps the $_{n}$ time point. By missing out the $_{n}$ time point this causes the numerical solution to drift apart as you step forward through the integration and as result generates the so-called computational mode. The simple leapfrog scheme is unstable but can be stabilised using the Robert-Asselin filter.

Le pfrog Sche e with Ro ert Asse in Fi ter

The Robert-Asselin filter was designed specifically for the leapfrog scheme in 1966 by Robert and in 1972 Asselin showed that it dampens the computational mode but leaves the physical mode relatively undamped [1][6][7]. Since then it has become known as the Robert-Asselin filter. After each leapfrog step, the filter mixes solutions from three consecutive time points at $_{n-1}$, $_{n}$ and $_{n+1}$ which can seen from Figure 2.1. The solution at the inner point at time $_{n}$ is displaced by

$$' = \frac{}{2}\left[x_{n-1} - 2x_n + x_{n+1}\right] \tag{2.2}$$

where is the filter parameter and the values x_{n-1}, x_n and x_{n+1} correspond to the time points $_{n-1}$, $_{n}$ and $_{n+1}$ respectively. Typically the filter parameter is taken to be 0.01. In Chapter 4 we will see the importance of the filter parameter .

The leapfrog scheme with the Robert-Asselin filter su□ers from num

Figure 2.1: Comparison between the (a) the standard Robert-Asselin filter and (b)

First Order

x_{n-2}. The third-order Adams-Bashforth method is an explicit scheme that requires one function evaluation per time step. The only potential problem is the storage requirements that prove to be a problem with all higher order schemes. Using the third-order Adams-Bashforth method eliminates the computational mode without introducing any other parameters [3].

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Mode Description

In this Chapter we will discuss the QUAGMIRE v1.3 model that will be used to test the behaviour of the time-di erencing schemes outlined in Chapter 2. We will look at the model equations along with the assumptions that have been made. Also, we will look at initialising the model to gather suitable initial conditions.

QUAGMIRE v

QUAGMIRE v1.3 is a quasi-geostrophic model that performs high-resolution simmay grow due to baroclinic instability. Baroclinic instability being an important mechanism that influences mid-latitude synoptic scale patterns that cause initial disturbances. These disturbances or perturbations are wave-like features that grow and decay with time.

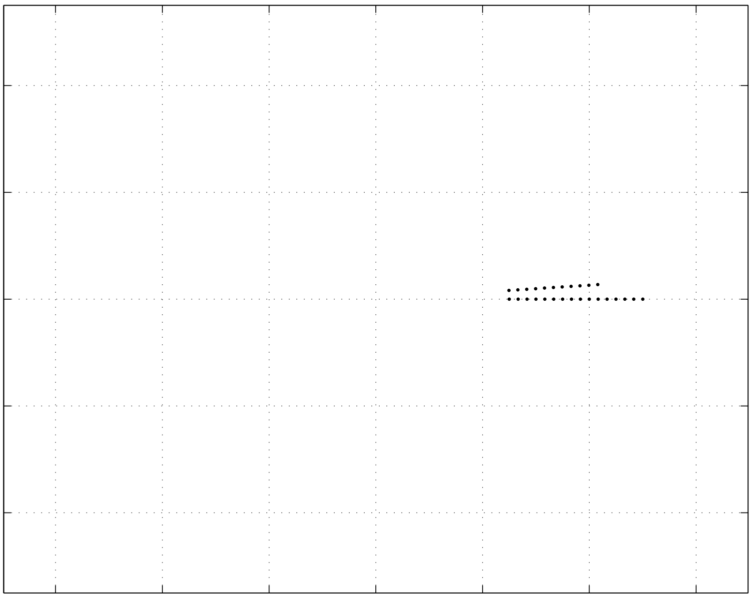
Figure 3.1: Two layer diagram showing the interface between the two layers where

$$\frac{\partial_2}{\partial} = \frac{1}{r}\frac{\partial_2}{\partial}\frac{\partial_2}{\partial r}\quad\frac{1}{r}\frac{\partial_2}{\partial r}\frac{\partial_2}{\partial}\quad\sqrt{\Omega r}.$$

Parameter	Meaning
$_1$	Stream function in the upper layer (layer 1)
$_2$	Stream function in the lower layer (layer 2)
$_1$	Potential vorticity in the upper layer (layer 1)
$_2$	Potential vorticity in the lower layer (layer 2)
Ω	Angular velocity
	Coriolis parameter (usually 10^{-}) for mid-latitudes)
	Scale height
	Acceleration due to gravity
$'$	Reduced gravity
$_1$	Kinematic viscosity in the upper layer (layer 1)
$_2$	Kinematic viscosity in the lower layer (layer 2)
r	Polar coordinate
	Polar coordinate
	Polar coordinate
$_1$	Perturbation potential vorticity in the upper layer (layer 1)
$_2$	Perturbation potential vorticity in the lower layer (layer 2)
∇	Laplacian operator

Table 3.1: Physical parameters used in the nonlinear model

The horizontal grid points are shown in Figure 3.2. For the mid-latitudes, normally $30^o - 60^o$N if the Northern hemisphere atmospheric jet stream is being represented or $30^o - 60^o$S if the oceanic Antarctic Circumpolar Current is being represented.



- **Atmosphere in hydrostatic balance.**
- **Ekman layer depths scale height H.**
- **Rossby number 1.**
- **Reduced gravity ′**

The first initial assumption is to assume incompressible fluids. This means when you

Initi is tion

The model uses a leapfrog time-stepping scheme with a Robert-Asselin filter. We may refer to this as the default time-stepping scheme. The Robert-Asselin filter is taken to be 0.01 and a time step $\Delta = 0.0008$ is used (in suitable units which are not of interest here). The model was run initially until the amplitude of the baroclinic

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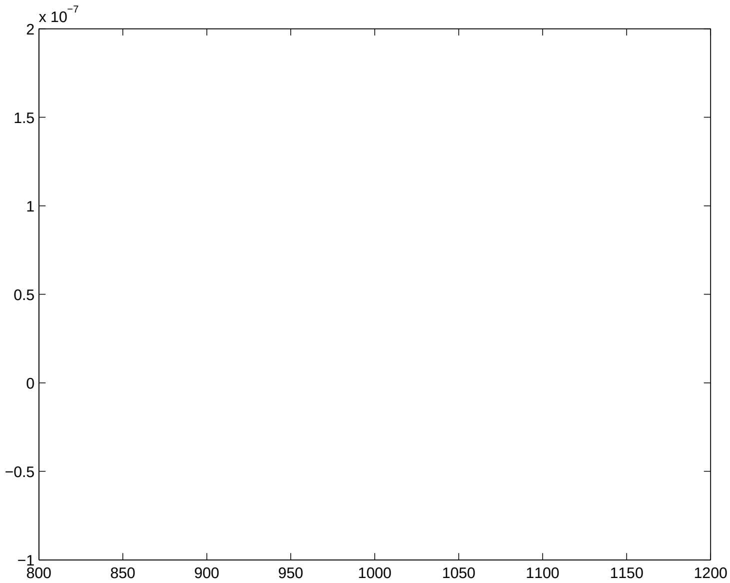
Nu eric Resu ts

In this Chapter we will compare the results from the time-di erencing schemes that we have implemented in the QUAGMIRE v1.3 model. We will look at the associated amplitude errors with the schemes.

Le pfrog with Ro ert Asse in Fi ter

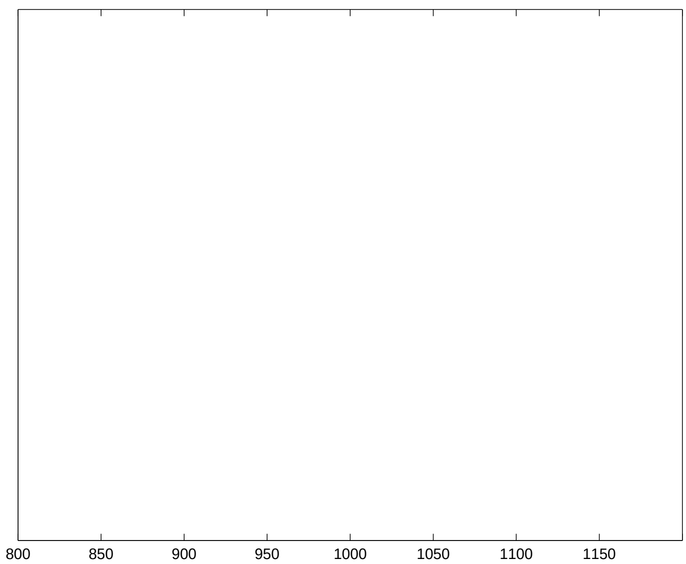
We will first begin with the default time-stepping scheme in the model by varying the filter paramater . By only changing it helps to determine the a ect the filter parameter has on the results. We will consider the standard filter and modified filters in turn.

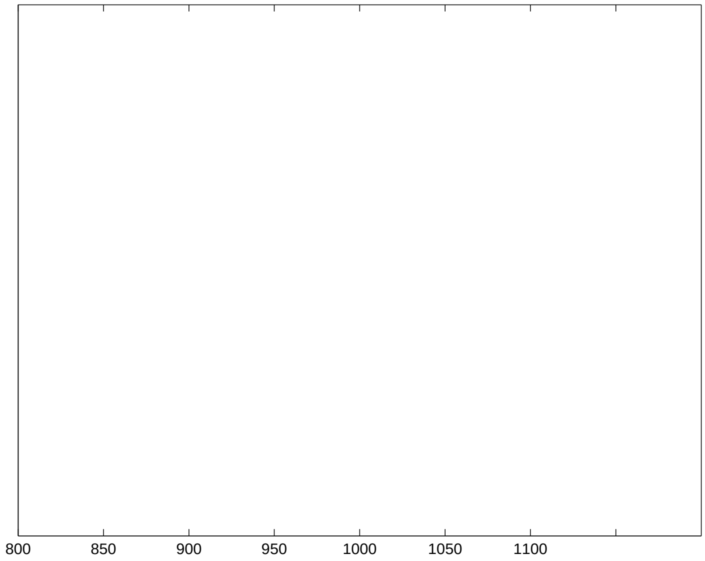
St nd ro


x 10^−7
2
1.5
1
0.5
0
−0.5
−1
800
850
900
950
1000
1050
1100
1150
1200

Modi ed Fi ter = 0

Finally, the modified filter with = 0 only displaces the outer point. This scheme is unconditionally unstable. Once 0.7 the model becomes unstable and terminates. This scheme is equal and opposite to the standard filter when = 1. The amplitude at the end of the integration decreases as increases. This can be seen from Figure 4.7 and 4.8. The ringing is apparent at the start of the time integration for the larger . Figure 4.9 shows the final amplitude decreasing as increases which does agree with the linear theory because equation 4.1 gives an amplitude error less than 1.

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850
900
950
1000
1050
1100

Ad s B shforth Sche es

We shall now consider the Adams-Bashforth schemes which we will run from the initial conditions set by the default scheme.

First Order

The first-order Adams-Bashforth scheme becomes unstable before the end of the

Second Order

We know the first-order Adams-Bashforth method is unstable so we shall now use the second-order method. Using the default time step Δ = 0.0008 the scheme is stable over the whole integration. Figure 4.11 shows as the time step increases the final amplitude also increases. The amplitude increases more rapidly for larger Δ . This is correct with the linear theory as the associated amplitude error suggests an increase in amplitude [3]. This is given by

$$|A| = 1 + \frac{\Delta}{4} \tag{4.3}$$

where Δ is the time step and is the angular frequency. We will derive the amplitude error in Section 4.3.

Deriving A p itude Errors

the amplification factor $|A|$ and the correct value [2].

The exact solution to equation 4.5 is

$$(\ ) = \quad (0) \exp(\quad \Delta \ ) \tag{4.17}$$

with an amplification factor of

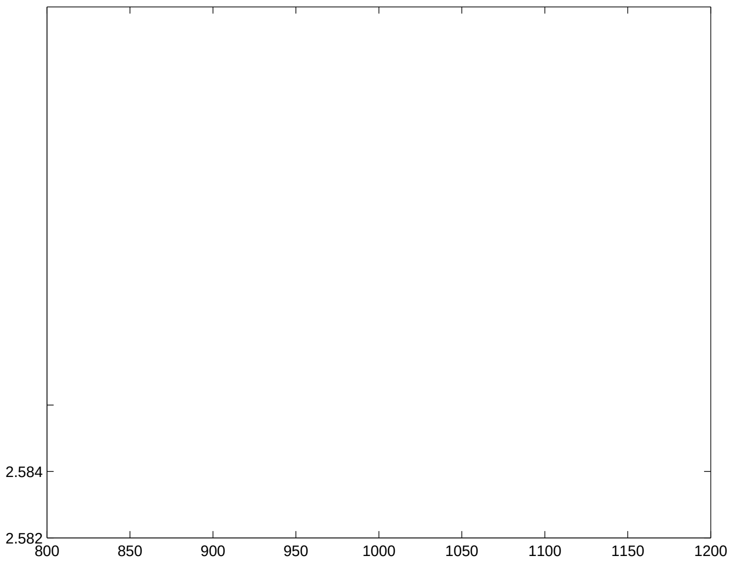
$$|A_{\mathrm{exact}}| = \exp(\quad \Delta \ ) \tag{4.18}$$

As we increase the numerical resolution by making $\Delta \ \rightarrow 0$ then $|A_{+}| \rightarrow 1$, $|A_{-}| \rightarrow 0$ and $|A_{\mathrm{exactact}}$

Co ined Le pfrog nd Forw rd Step

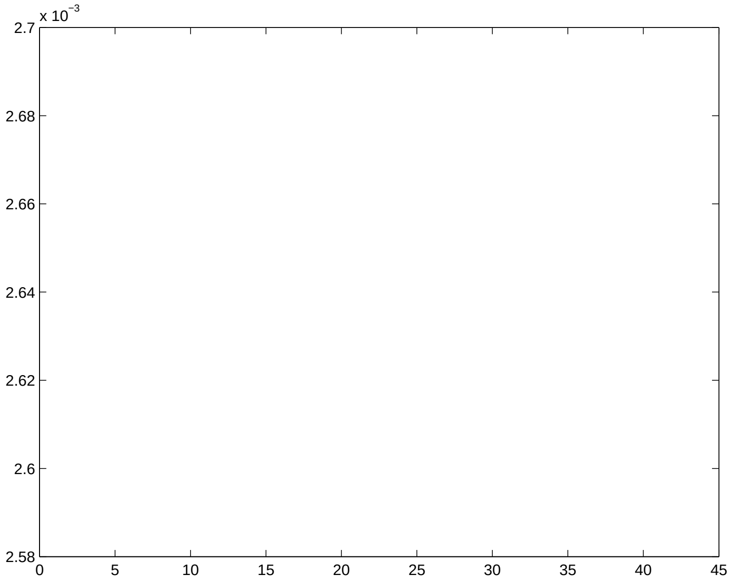
We have so far implemented a leapfrog with a Robert-Asselin filter and a first-order Adams-Bashforth method separately.

We will now combine the leapfrog scheme and a simple forward scheme to see


2.584
2.582
800
850
900
950
1000
1050
1100
1150
1200

Ch pter

Sensitivity Tests


x 10^-3
2.7
2.68
2.66
2.64
2.62
2.6
2.58
0
5
10
15
20
25
30
35
40
45

When taking the forcing and dissipation from the model and applying only advection the oscillations become more irregular and the wave amplitudes do not grow as large. Figure 5.3 shows that the amplitudes of the three case of perfectly map each other until the final stages of the integration. At this point the phase and amplitudes of the waves begin to deviate.

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Conc usion

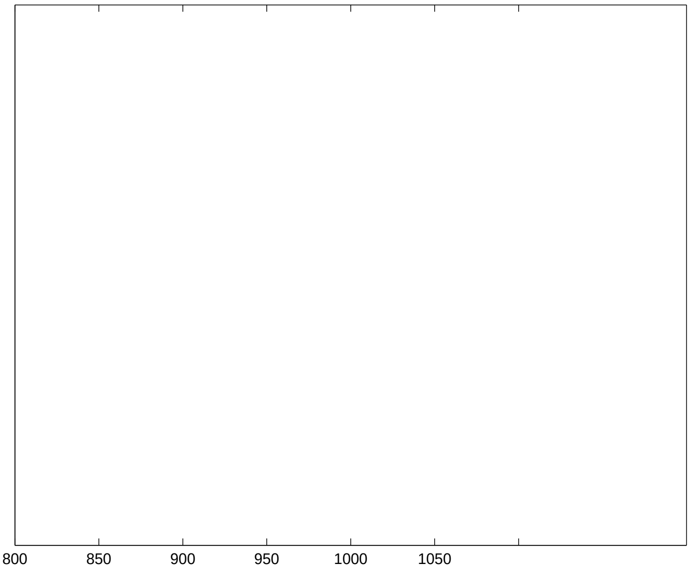
In this final Chapter we shall conclude the dissertation by summarising the results from the QUAGMIRE v1.3 model from all the different time-differencing schemes used throughout. The benefits and costs of the schemes for NWP models and the future work that needs to be undertaken to take this area of Meteorology to the next level.

Su ry

The aim of this dissertation was to implement a number of time-differencing schemes in a simple nonlinear numerical model and compare the time step errors in the schemes with the predictions of simple linear analysis. Then finally deciding whether the schemes could be implemented into existing NWP models.

We have implemented a number of time-differencing schemes into the nonlinear QUAGMIRE v1.3 model. The development of baroclinic waves in the model indicate a good resemblance between the model and laboratory. This indicates that the

was then tested for the three cases of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Future ork

In the future it would be beneficial to implement and employ more time-di□erencing schemes into the QUAGMIRE v1.3 model to understand a wider concept of possible schemes that could be used in NWP models. Above all, not every scheme will be perfect, but by analysing the benefits and costs of the schemes it may be possible to determine if the scheme should and could be employed into existing models. If the benefitsd59320E0.217.4014(e)0.0490113]TJΩ-390.464-18.7828TdΩ[(p)-27.4014(e)0.0495218(r)-.7.4014

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