Aperiodic dynamics in a deterministic model of attitude formation in social groups

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Abstract

Homophily and social influence are the fundamental mechanisms that drive the evolution of attitudes, beliefs and behaviour within social groups. Homophily relates the similarity between pairs of individuals' attitudinal states to their frequency of interaction, and hence structural tie strength, while social influence causes the convergence of individuals' states during interaction. Building on these basic elements, we propose a new mathematical modelling framework to describe the evolution of attitudes within a group of interacting agents. Specifically, our model describes sub-conscious attitudes that have an activator-inhibitor relationship. We consider a homogeneous population using a deterministic, continuous-time dynamical system. Surprisingly, the combined effects of homophily and social influence do not necessarily lead to

1. Introduction

Our attitudes and opinions have a reciprocal relationship with those around us: who we know depends on what we have in common, while simultaneously our beliefs influence, and are influenced by, those of our peers. These two mechanisms—homophily and social influence—underpin a wide range of social phenomena, including the diffusion of innovations [1, 2, 3, 4, 5], complex contagions [6, 7, 8], collective action [9, 10, 11], opinion dynamics [12, 13, 14, 15, 16, 17, 18, 19, 20] and the emergence of social norms [21, 22, 23]. Thus homophily and social influence represent the atomistic ingredients for models of social dynamics [24]. Starting with these basic elements, we investigate a new type of modelling framework intended to describe the coevolution of sub-conscious attitudinal states and social tie strengths in a population of interacting agents.

The first ingredient in our modelling framework, homophily, relates the similarity of individuals to their frequency of interaction [25]. Thus hoevolution is driven by homophily. Although our model is built on the notions of homophily and social influence described above, we point out that differentiating between the effects of these processes, particularly in observational settings, may be very difficult [35, 36].

Social scientists have developed 'agent-based' models that incorporate homophily and social influence in order to examine a variety of social-phenomena, including group stability [37], social differentiation [38] and cultural dissemination [39], where a culture is defined as an attribute that is subject to social influence. In such models, an agent's state is typically described by a vector of discrete cultures and the more similar (according to some metric) t.974098(g)-2.83755(s)4.57545
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introduction, we shall more specifically assume that (1) is drawn from a class of activator-inhibitor systems.

Now suppose that the individuals are connected up by a dynamically evolving weighted network. Let A( ) denote the $\times$ weighted adjacency matrix for this network at time , with the th term, A_{ij}( ), representing the instantaneous weight (frequency and/or tie strength) of the mutual influence between individual and individual at time . Throughout we assert that A( ) is symmetric, contains values bounded in [0,1] and has a zero diagonal (no self influence). We extend (1) and adopt a first order model for the coupled system:

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) + D^{ND\ T} \quad {}^{T} \quad {}^{T}$$

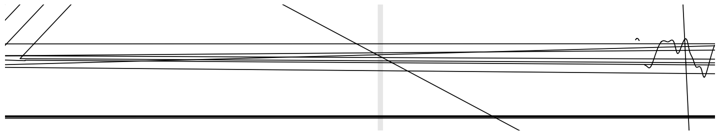
between choices of D and $\mathbf{df}(\mathbf{x}^*)$, where there is a *window of instability* for an intermediate range of λ, is know as a Turing instability. Turing instabilities occur in a number of mathematical applications and are tied to the use of activator-inhibitor systems (in the state space equations, such as (1) here), where inhibitions diffuse faster than activational variables.

Now we can see the possible tension between homophily and Turing instability in the attitude dynamics when the timescale of the evolving network, , is comparable to the changes in agents' states. There are two distinct types of described here.

3. Examples

We wish to consider activator-inhibitor systems as candidates for the attitudinal dynamics in (1) and hence (3). The simplest such system has = 2, with a single inhibitory variable, ( ), and a single activational variable, ( ). Let $\mathbf{x}_i$( ) = ($_i$( ))

with the eigenvalues of the Laplacian λ_i. When one or more of the λ_i



are dynamic and their evolution is driven by homophily. In some sense, the corresponding elements of our model are like a continuous-time version of the Flache and Macy model. However, Flache and Macy consider the weights

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