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Two Maps and Worldwide Ipod Interest

by

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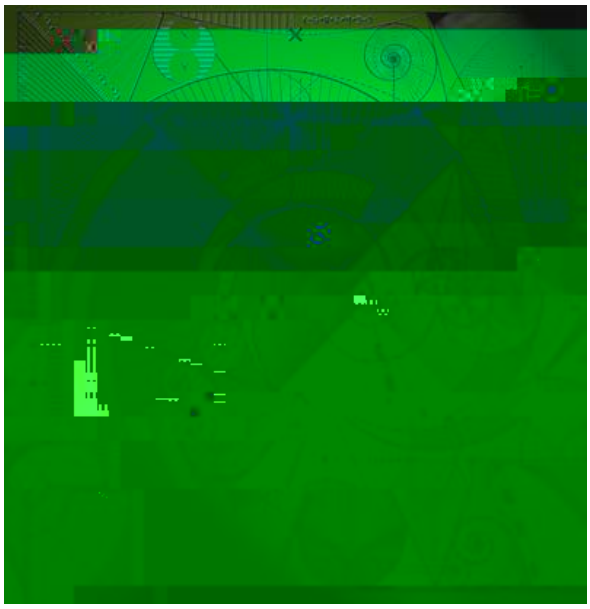

Two Maps and Worldwide Ipod Interest

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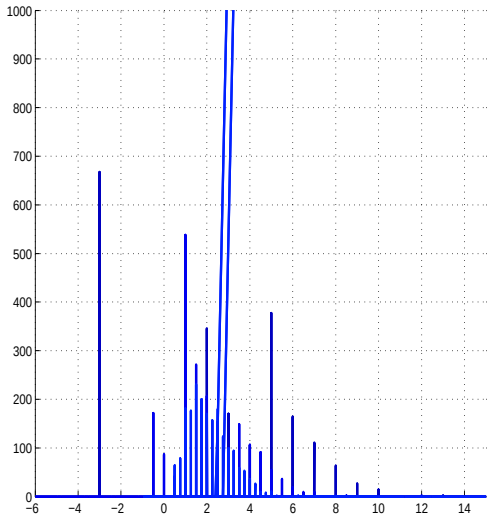
Abstract. There is often a desire to determine if the dynamics of interest are chaotic or not. Since positive Lyapunov exponents are a signature for chaos, they are often used to determine this. Reliable estimates of Lyapunov exponents should demonstrate evidence of convergence; b(n)4.81675c7.159(e)29.14.81944(a)2 8 0 T1672(n)4.81944(t)-39282(h)4.81672(a382(sr)-309.402

initial distribution $\rho_0(x$ so that after N iterations $\rho_N(x$ is an estimate of the invariant distribution. One would then sample initial states from $\rho_N(x$ to estimate λ_N's. The aim here is to sample the initial states according to the invariant measure, if it exists. As a consequence of Oseledec's theorem [4], the distribution of the λ_N's will converge to a delta distribution centred at Λ as $N \rightarrow \infty$. The aforementioned approach is used in the numerical computations of the two maps in the next section.

It is ill-advised to use a single initial state to estimate Λ, even if the graph of λ_N versus N appears to approach a horizontal line. When one is faced with real data, the approach of the previous paragraph cannot be used unless there is a reliable mathematical model of the system. Nonetheless, one may use bootstrap approaches suggested in [10] for various values of N to assess convergence in the distribution of the λ_N's. In the ipod example, we consider only the maximal Lyapunov exponent and compute each λ_N as discuessed in [11].

THE TWO MAPS

Two maps and numerical computations of their Lyapunov exponents are discussed in this section. There is clear convergence of distributions of finite time Lyapunov exponents in the first map, bh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this study. For further analyses, we discarded the time series prior to 2005 as transient behaviour. In order to compute distributions of finite time Lyapunov exponents, there are two parameters that have to be determined first. One of the parameters to determine is an appropriate dimension for embedding the scalar time series into higher dimensions. In order to determine the dimension, we first need the time delay. We selected the time delay as discussed in [14] and the dimension by the method of false nearest neighbours discuss