Early Warning with Calibrated and Sharper Probabilistic Forecasts

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Abstract

Given a nonlinear deterministic model, a density forecast i

ar na ca brat on w. . n , not a . o, at t. sa t D ns t or asts ar sa, to b probab st a a brat, an on t. I s ar un or , str but, arF na a brat on r rs to t. as w. n t. t av raF o a , ns t or asts s qua to t. at o , a or asts . r s no p r a wa o ass ss nF x , an a brat on .

Gn t nF *t a* t. n on tur, t. at w. n a subs t o t. s o, o a . brat on . o s t. n t. pr , t v , str but ons ar at ast as spr a out as t. , a or asts w. . on tur t. t r , a *s, arpn ss pr nc p* t r *s, arpn ss* s s to . av b n o n, b Bross an t s a asur o . ow on ntrat, probab st or asts ar an a prop rt o o boF

to t s attr but s w. m ato oF s n u , m o s o a brat on ass ss , ar probab st an m arF na a brat on w. t m p as s , t at ntrop s ou , b us , to m asur s arpn ss n w pro pos t on t at a , r ss s Gn t nF *a* s on tur ps xp a n t n nFs r as Bro r m t g s r t s to not n u nF a n un t ons n t , ns t st m at on was ts Fuarant , n r as n var an o t pr , t v , str but on m par , to t raw ns m b w , m onstrat t at t us o var aw to m asur s a pn ss ou , b m s a nF s pap r s or Fan s , as o ows g n xt s t on , s uss s or ast qua t s t at ar m u at v m asur , b t oFa t m s or nF ru In part u ar a , o pos g t on o t s s or nF ru s pr s nt , s arpn ss pr n p on tur , b Gn t nF *a g* s , s uss , an a r vant propos t on pr s nt , n § In § t m t o , o oF m p o , to pro u g ns t or asts s out n , su ts on rn nF , ns t or asts obta n , v a t oFar t m s or nF ru w t g sp t to a non n ar tron r u t a , s uss , n § t on F v s on u nF r m ar s App n s A an B onta n t proo o t propos t on on rn nF t s arpn ss pr n p an app n x C onta ns proo s or t r st o t propos t ons

2 Probabilistic-Forecast Quality

o m ssp at on p a s m tat ons on t va u o probab st or asts n t ot r an onsi m rs o pr asts m a , m an pr , t w , str but ons t at ar bot *ca brat* an *s arp* I su or asts ar ssu at onF t m or ons t n ar varn nF s a or , su F st t at t s qua t s an b u at quant g b t oFar t m s or nF ru propos b Goo , r ar ot r s on nF ru s ava ab or s t on s Gn t nF a t r For nstan t r s t Br r s on Br r , s ow v r , o pos s nto m an t m s urp g , so o w ar not vant to our , s uss on an t s su tab or at For a v nts A F n ra sat on o t Br r s or to , ns t or asts s t

o two Gauss ns w av ow r ntrop t arva s nF Gauss an str but on o t sp
var an u m or a str but on o a F r var an an av a ow r ntrop t an
t at o ow r var an

arpn ss as a so b n quant b on n nt rva s a t r *a*
Gn t nF *a* Con n nt rva s s ar a s m ar w a n ss to var an n t
s ns t at a b m o a str but on t at s a r on ntrat on t two m o s an av
ar F r on n nt rva s t an a um m o a str but on t at s a r spr a out A so
F v n two non s mm tr str but ons w o t m s m s arp r ou p n
on w at t on n v s

2.3 Calibration

a brat on o ns t or asts s a w g tro n sub t u o t t ratur
ta s t stan t at a a brat or ast nF s st m s tanta ount to a orr t sp
m o Corra wanson prov a o pr ns v surv o om a stat st a g
t n qu s or ass ss nF a brat on o ns t or asts to t m n t un r nF
m o s gorr t sp wor o Gn t nF *a* str s a s or b
prov nF a a brat on ra wor t at a m o at s m o m ssp at on
bro own a brat on nto t r m o g s a o w ou b ass ss s parat
uppos a probab t g or ast nF s st m ssu s pr t v str but ons $\{ _t x \}_{t=1}^{T}$
w t ata F n rat nF wro ss ssu s a or asts $\{ _t x \}_{t=1}^{T}$ Gn t nF *a*
t n n t o ow nF m o s o a brat on

- s qu n $\{ _t x \}_{t=1}^{T}$ s *probab st ca y ca brat* r at v to $\{ _t x \}_{t=1}^{T}$

$$\quad \sum_{t=1}^{T} \; _t\{ _t^{-1} p \} \quad p \quad p \square$$

- s qu n $\{ _t x \}_{t=1}^{T}$ s *c anc ca brat* r at v to $\{ _t x \}_{t=1}^{T}$

$$\quad \sum_{t=1}^{T} \; _t^{-1}\{ _t x \} \quad x \quad x \square \square$$

- or ast r s *ar na y ca brat*

$$m \quad \sum_{t=1}^{T} \; _t x \qquad m \quad \sum_{t=1}^{T} \; _t x$$

I w av a t m s r s o obs rvat ons x_t t n $_t \quad _t x_t$ s a *probab ty nt a trans or* I Corra wanson D bo *a* 99 n o m t o t I s
s qu a nt to probab st a brat on Gn t nF *a* A v ua nsp t on o
I stoFra s wou r v a obv ous partur s ro un o m t un r nF
m o s orr t sp an on $_t \square$

uppos w av a m s r s o ns t or asts $\{ _t x \}_{t\ 1}$ n n t or ast r s m ato oF as

$$_T x \quad \sum_{t=1}^{T} {}_t x$$

n a or ast r w o ssu s

$$_t x \quad \overline{}_T x \quad \sum_{t=1}^{T} {}_t x$$

or a $\{$ $\}$ to b t *n t c ato o ca or east* I $_t x$ m $\overline{}_T x$ t n w av t *c ato o ca or east* A or ast r s *n t ar na y ca brat* $_T x \quad \overline{}_T x$

For a pra t a purpos s sy n t an w av no a ss to t $_t x$ s H n t s u t to ass ss n t m arF nag a brat on I $_T \quad _T \approx$ w r s sm m tr t n w an ta to b arF nouF to va uat m arF na a brat on o t s n w an us t H nF r stan o ar an o put

$$_T \quad _c$$

w an or ns t or ast st at s o t or

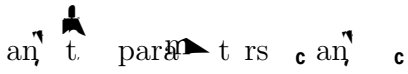
$$ {}^{(t)} x \quad \frac{}{} \sum_{i=1}^{N} \left\{ \left(x - X_i^{(t)} - \right. \right. \quad \right\} $$

w r an ar r sp t v ban w t an o s t para t rs os n a or nF to past r or an an s t rn un t on ns t or ast n rs ro t tra t ona ar n st at s b t o s t para t r It s s ar to t Ba s an o Av raF propos b a t r a w a m or b as orr t on an qua w F ts H r t ns b m b rs ar x anF ab an o not r pr s nt st n t o s t nF us nF v r an o s not a ount or m o m ssp at on

o a ount or m o m ssp at on t us rst not a r or o past t s r s an rr spon nF ns b or asts b V_T $\{ \quad {}_t X^{(t)} \}_{t=1}^{T}$ n t ns t or asts w os para t rs an a s t b ta nF nto a ount past p r or m an m a b not b ${}^{(t)} x|V_T$ ${}^{(t)} x|V_T$ as t sa or as n ts para t rs ar s t b o nF t m n m sat on

$$ \min_{\sigma>0,\mu} \left\{ -\frac{}{} \sum_{t=1}^{T} \text{oF} \; {}^{(t)} \quad {}_t|V_T \right. $$

v n r rta n assu pt ons o nF t m n m sat on n s



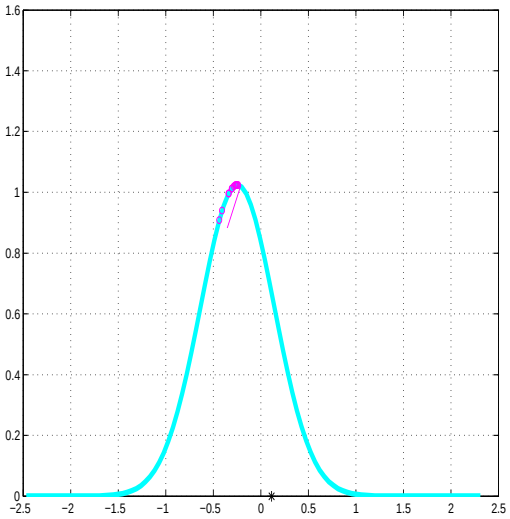
w. n g D sp t t. s r, u t on s m xtur or asts m a st b ss s. arp t. an m ato oF n t. s ns o ntrop A stra F t orwar, app at on o t. Ku ba L b r an, J ns n s n qua t s a, s to t r at ons

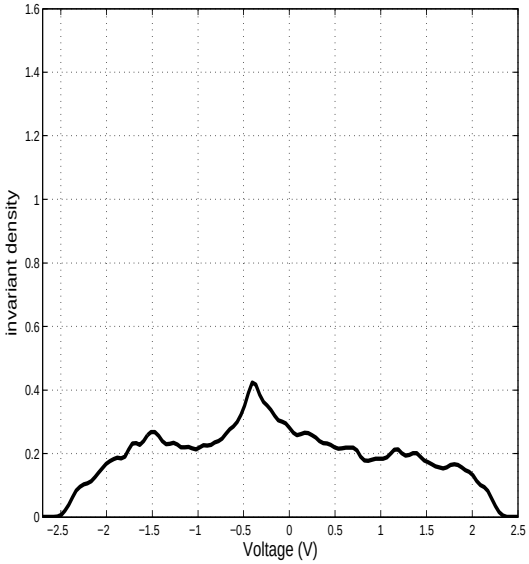
$$\{ {}^{(t)}\} - \quad {}_{c} \leq \{ {}^{(t)}\} \leq \{ {}^{(t)}\} - \{ {}^{(t)} \quad {}_{c}\}$$

$$- \{ {}_{c} {}^{(t)}\} - {}_{c}$$

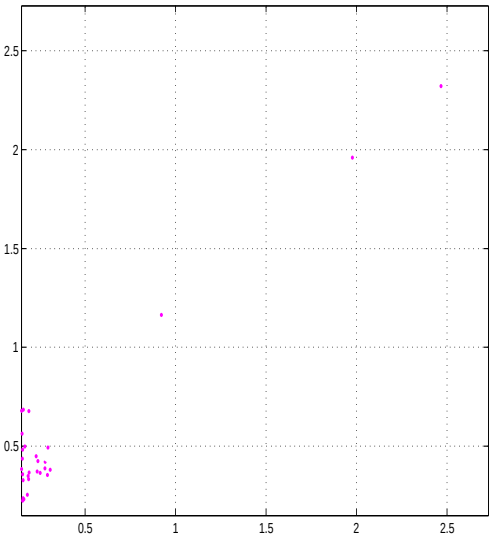
w. r $- \int x$ oF x, x an $- \int x$ oF x, x ar t. ntrop an, ross ntrop r sp t v r or t. n ssar an, su nt on, t ons or $\{ {}^{(t)}\} \geq {}_{c}$ to o, ar t. at ${}_{c} \setminus \{ {}^{(t)} {}_{c}\}$ an ${}_{c} \setminus \{ {}^{(t)}\}$ r sp t v n v r t. m ato oF s s. arp r t. an t. m xtur or ast t s. ou, b ssu, as t. or ast nst a o t. m xtur

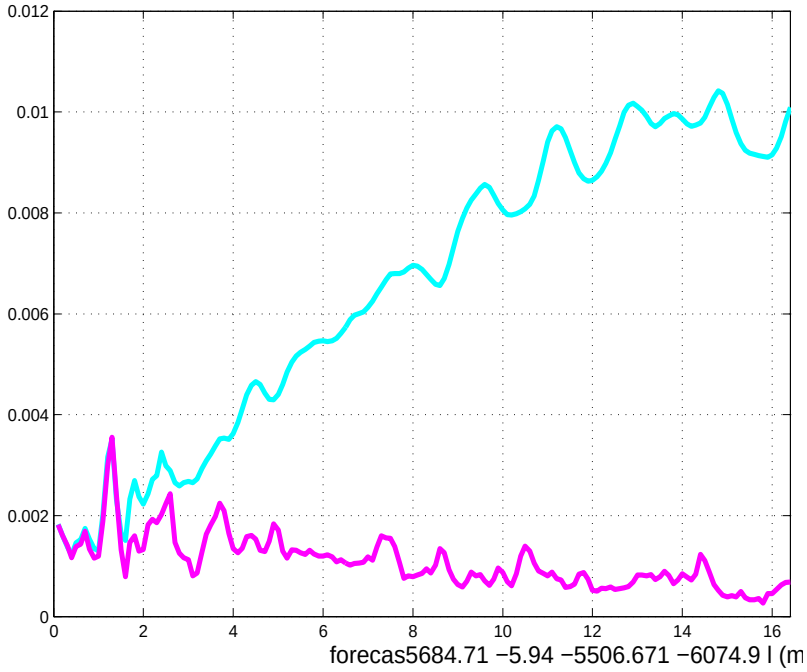
It s not obv ous w at t t o t. m xtur s on a brat on x pt t. at n u, nF m ato oF m prov s t. KL, stan r o t, a or asts v rt. ss t. m xtur para t r t. at m m s s t. oFar t. m s or , s t. quat on

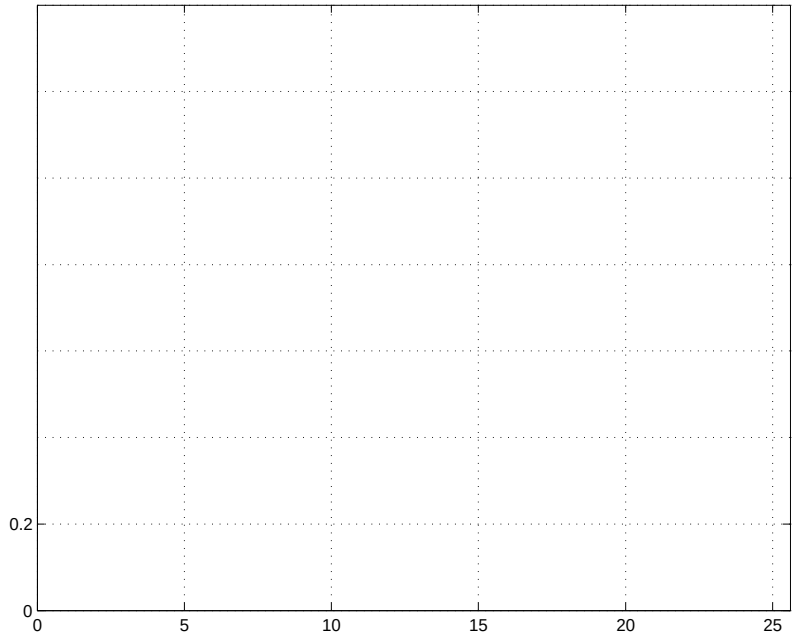

1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−2.5
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
2.5

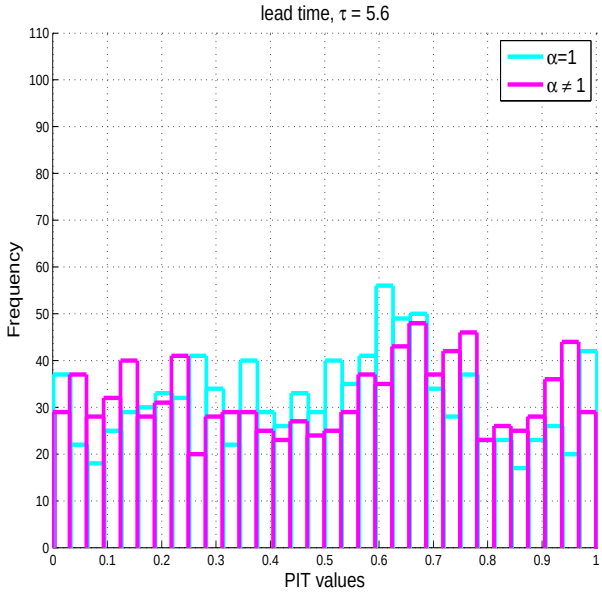


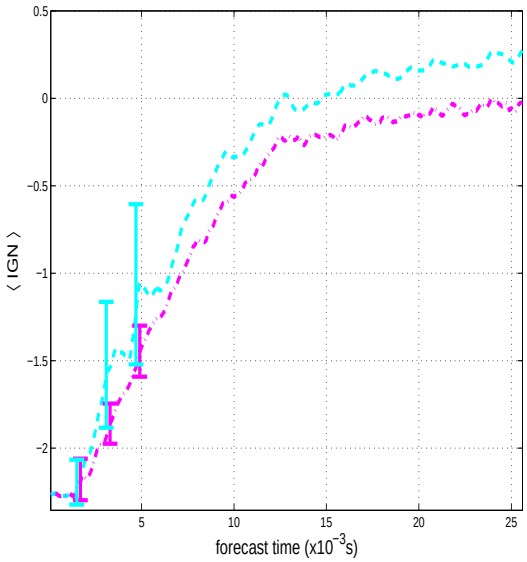
F Fur *Graph of the climatology of the circuit estimated from data. Its entropy is 2.15, which is greater than the entropies shown in figure 1.*

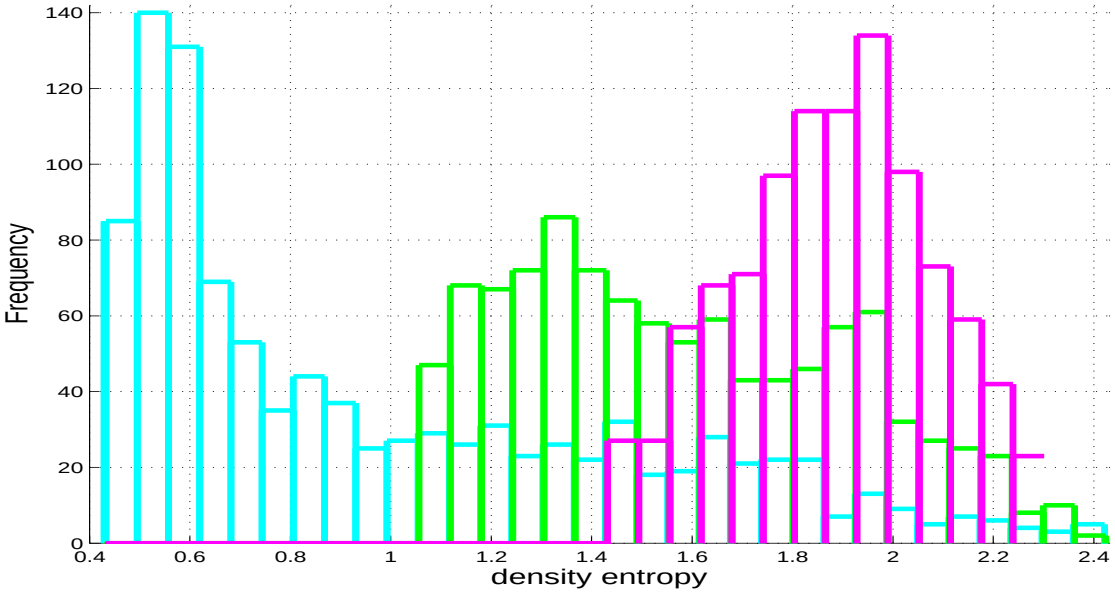



0.012
0.01
0.008
0.006
0.004
0.002
0
0
2
4
6
8
10
12
14
16
forecas5684.71 −5.94 −5506.671 −6074.9 l (m


0.2
0
0
5
10
15
20
25


lead time, τ = 5.6
α=1
α ≠ 1
Frequency
PIT values
0
10
20
30
40
50
60
70
80
90
100
110
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


0.5
0
−0.5
−1
−1.5
−2
5
10
15
20
25
⟨ IGN ⟩
forecast time (x10^{-3}s)


Frequency
density entropy
0
20
40
60
80
100
120
140
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4

annon CE tor *Co un cat on n t pr s nc o no s* vo ro

$_{t}\ x \qquad {}_{t}^{-1}\ {}_{1}\{\ {}_{1}\ x\ \}$ It t n o ows t at

$$\sum_{t=1}^{T}\ {}_{t}\left[\ {}_{t}\left\{\ {}_{s}^{-1}\ x_{s}\ \right\}\right]$$