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Flow-Dependent Balance Conditions for Incremental Data Assimilation: Elliptic Operators

by

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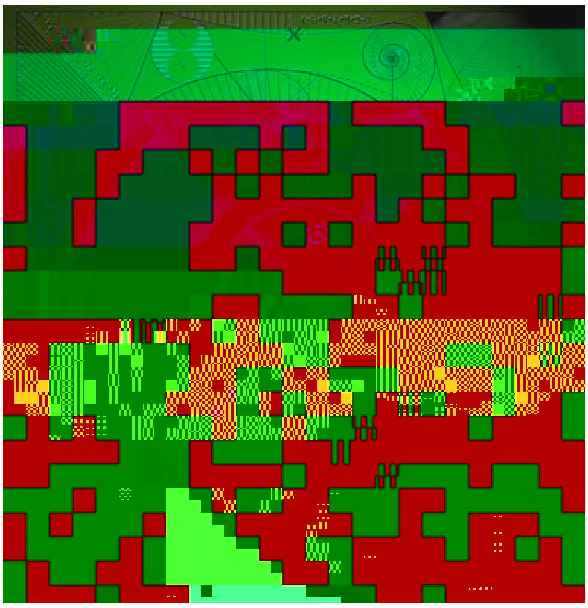

(L op. cit

et al., 1 2)

(1).

2 B B

2.1, 2.2, 02. 2.

2.1 A B

et al. (2000).

$(u; v; p;) \mapsto (; ; {}^{A}p)$,

$$\equiv \cdot r \times r^2 ; \qquad (1)$$

$$\equiv r \cdot r^2 ; \qquad (1b)$$

3.2

$$u_g = -\frac{g}{af}\frac{@h}{@}; \qquad v_g = \frac{g}{af}\frac{@h}{@}: \qquad (1\ )$$

fi ^{b},

$$^{b} \equiv \ - \frac{}{f}(\ \cdot r)(\ \times\ );$$

fi

$$u^b \equiv u_g - \frac{}{f}\left(\frac{u_g}{a}\frac{@v_g}{@} + \frac{v_g}{a}\frac{@v_g}{@} + \frac{}{a}u_g^2\right); \qquad (1\ )$$

$$v^b \equiv v_g + \frac{}{f}\left(\frac{u_g}{a}\frac{@u_g}{@} + \frac{v_g}{a}\frac{@u_g}{@} - \frac{}{a}u_g v_g\right); \qquad (1\ )$$

, (12). (1) (1) (1),

$$u^b = -\frac{g}{af}\frac{@h}{@} - \frac{g^2}{a^3 f^3\ ^2}\left(\frac{@h}{@}\frac{@^2h}{@\ @} - \frac{@h}{@}\frac{@^2h}{@\ ^2} + \left(\left(\frac{@h}{@}\right)^2 + \ ^2\left(\frac{@h}{@}\right)^2\right) - \frac{}{f}\left(\frac{@h}{@}\right)^2\right); \qquad (1\ )$$

$$v^b = \frac{g}{af}\frac{@h}{@} + \frac{g^2}{a^3 f^3}\left(\frac{@h}{@}\frac{@^2h}{@\ @} - \frac{@h}{@}\frac{@^2h}{@\ ^2} + \frac{@h}{@}\frac{@h}{@} + \frac{}{f}\frac{@h}{@}\frac{@h}{@}\right); \qquad (1\ )$$

$$\equiv \frac{df}{d}: \qquad (1\ )$$

fi

h

$$
\begin{aligned}
{}^{b} \quad & \frac{g}{f} r^{2} h + \frac{g}{f^{2}a^{2}} \frac{\partial h}{\partial} + \frac{2g^{2}}{f^{3}a^{4} \quad {}^{2}} \left(\left(\frac{\partial^{2} h}{\partial \ \partial} \right)^{2} \right. \\
- \quad & \frac{\partial^{2} h}{\partial \ {}^{2}} \frac{\partial^{2} h}{\partial \ {}^{2}} + 2 \qquad \frac{\partial h}{\partial} \frac{\partial^{2} h}{\partial \ \partial} + \quad {}^{2} \left(\frac{\partial h}{\partial} \right)^{2} \\
+ \quad & \qquad \frac{\partial h}{\partial} \frac{\partial^{2} h}{\partial \ {}^{2}} \\
+ \quad & \frac{1}{2} \left(\left(\frac{\partial h}{\partial} \right)^{2} + \quad {}^{2} \left(\frac{\partial h}{\partial} \right)^{2} \right) \\
+ \quad & \frac{}{f} \left(2 \frac{\partial h}{\partial} \frac{\partial^{2} h}{\partial \ {}^{2}} - 2 \frac{\partial h}{\partial} \frac{\partial^{2} h}{\partial \ \partial} - 2 \qquad \left(\frac{\partial h}{\partial} \right)^{2} \right. \\
+ \quad & 2 \frac{}{f} \left(\frac{\partial h}{\partial} \right)^{2} - \frac{}{2} \qquad \left. \left(\frac{\partial h}{\partial} \right)^{2} \right) \\
- \quad & \left. \frac{1}{2f} \frac{\partial}{\partial} \left(\frac{\partial h}{\partial} \right)^{2} \right) : \qquad (1 \ )
\end{aligned}
$$

(1) h

$$\frac{\partial^2 h}{\partial \quad ^2}; \qquad \frac{\partial^2 h}{\partial \quad ^2}; \qquad \frac{\partial^2 h}{\partial \quad \partial}: \tag{22}$$

ffi A, B, C, D E (21) w (; ; $h; p; q$), p $\partial h/\partial$; q $\partial h/\partial$.

fi (1) (20)

, fi 1 > 0,

(20) 0 .

, (20), ffi

(1), fi A.

A, (20)

$$\frac{(1+2\ )f}{2}+\frac{2}{fa^2}\left(v_g^2+u_g^2\right)-\frac{}{fa}u_g+\frac{2}{f^2a^2}u_g^2$$

$$+\left(\frac{2\ \ 2}{f^3a^2}-\frac{@}{@}\frac{2\ \ 2}{f^2a^2}\right)v_g^2 > hq^b\ \ (f+\ ^b):$$

f

v .

v . v h

, .

fi fi , v

h , , $@h=@$ 0. w

h h . / 10 1 (. (2)-10)-2 ( )

, (.)-2 ( )- 0 (

(.)- ( )- -

v w (1) (20), w

.

4 .

(.) fi

fi , ^{b}

(1) 20).

.

h^0

$$
\begin{aligned}
b0 \equiv{}& \frac{g}{f} r^2 h^0 + \frac{g}{f^2 a^2} \frac{\partial h^0}{\partial} + \frac{2g^2}{a^4 f^3}{}^2 \Big(2 \frac{\partial^2 \bar{h}}{\partial \, \partial} \frac{\partial^2 h^0}{\partial \, \partial} \; 2 \quad 2h \, \partial \\
& - \frac{\partial^2 \bar{h}}{\partial^2} \frac{\partial^2 h^0}{\partial^2} - \frac{\partial^2 \bar{h}}{\partial^2} \frac{\partial^2 h^0}{\partial^2} + 2 \frac{\partial^2 \bar{h}}{\partial \, \partial} \frac{\partial h^0}{\partial} \\
& + 2 \frac{\partial \bar{h}}{\partial} \frac{\partial^2 h^0}{\partial \, \partial} + 2 \; {}^2 \frac{\partial \bar{h}}{\partial} \frac{\partial h^0}{\partial} + {}^2 \frac{\partial \bar{h}}{\partial} {}^0
\end{aligned}
$$

$\frac{\partial h}{\partial}$

4.2 [illegible]

5 A

(0) (1)

2-

et al., 1 2).

(1).

$$h = \frac{1}{g}\left(gh_0 + a^2A(\ ) + a^2B(\ )\ \ R + a^2C(\ )\ \ 2R\ \right);$$

$$u = a!\ \ + aK\ \ ^{R-1}\left(R\ \ ^2 - \ \ ^2\right) \times\ \ R\ ;$$

$$v = -aKR\ \ ^{R-1}\ \ R\ ;$$

$$= 2!\ \ - K\ \ ^{R}\left(R^2 + \ R + 2\right) \times\ \ R\ ;$$

h_0 $A(\ ); B(\ )$ $C(\ )$

$$A(\ ) = \frac{!}{2}(2\Omega + !)\ \ ^2 + \frac{1}{\ }K^2\ \ ^{2R}\ (R+1) \times\ \ ^2 + \left(2R^2 - R - 2\right) - 2R^2\ \ ^{-2}\ ];$$

$$B(\ ) = \frac{2(\Omega + !)K}{(R+1)(R+2)}\ \ ^{R}\left[\left(R^2 + 2R + 2\right) - (R+1)^2\ \ ^2\right];$$

$$C(\ ) = \frac{K^2}{\ }\ \ ^{2R}\left[(R+1)\ \ ^2 - (R+2)\right];$$

$!$, K R , h_0.

R , ,

(1). h_0,

, $!$

K .

ff , B_u. v

$$B_u \equiv \frac{\sqrt{gh}}{fL} = \frac{L_R}{L}; \qquad (\ )$$

L L_R -

.

ff v B_u ff .

(2002).

fi , 1 , $h_0 =$ 0m ! =

$K =$

b ‒ ‒ ffi B^2 .

ffi (s^{-6})	1	2	
$\frac{1\ g^2 \left(\frac{@\bar{u}_g}{@}\right)^2}{\bar{h}^2}$	$2{:}\ \times 10^{-14}$	$:\ 1 \times 10^{-16}$	$:\ \times 10^{-18}$
$-\frac{2g^2\ \frac{@\bar{u}_g}{@}\bar{v}_g}{a\bar{h}_2^2}$	$-2{:}2\ \times 10^{-14}$	$-\ :\ 2 \times 10^{-16}$	$-\ :\ 2 \times 10^{18}$
$\frac{1\ g^2\ 2\ \bar{v}_g^2}{a^2\bar{h}^2}$	$:1\ \times 10^{-15}$	$1{:}\ \times 10^{-16}$	$1{:}\ 2 \times 10^{-18}$
B^2	$:\ \times 10^{-15}$	$1{:}\ 2 \times 10^{-16}$	$:\ \times 10^{-18}$

, 1 10^2 ffi AC . v , 2 10 . v 2 L ff b .

h_{11} / :11 10 . 1 0 2 (`) .1 3: :11 () / 1 10 . -. 0 (1 -2

ffi AC

ffi (s^{-6})	1	2	
$\frac{g^2 f^2}{\bar{h}^2}$ 2	: $\times 10^{-11}$	2: $\times 10^{-14}$	:11 $\times 10^{-14}$
$\frac{g^2 f}{\bar{h}^{20}}$ $\frac{@\bar{u}_g}{@_{11}}$	h_{11}	0_1	

$8g^{2}$ @ @813(i.56.15 6.74 Td[(@)]TJ /F8 99 T22)1.1388 0 Td4.882u4[(8960[(2674 Tf 5.75[.875TJ /F8 9.963 Tf 6.1363 T60[(-40 Td6.0

ffi ff

ffi (ms^{-3})		1	2	
gf	$h_{\lambda\lambda}$	$1{:}01 \times 10^{-3}$	$1{:}01 \times 10^{-3}$	$1{:}01 \times 10^{-3}$
fg	h	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$
$^2\ fg$	$h_{\theta\theta}$	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$
$2g\frac{@\bar{u}_g}{@}$	$h_{\theta\lambda}$	$2{:}\ 2 \times 10^{-5}$	$2{:}2 \times$ [illegible]	10^{-5} [illegible] $\times 10^{-5}$
$2g\ @\bar{v}_g h$	$:0 \times 10^{5}$	$2{:}0 \times$ [illegible]	[illegible]	[illegible]

.74 7io.P d[( )]TJ /F15 6.974 Tf 6.23 0 Td[(5)]TJ

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