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Four-dimensional variational data assimilation for high resolution nested models

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Abstract

Four-dimensional variational data assimilation (4D-Var) is used in environmental prediction to estimate the state of a system from measurements. When 4D-Var is applied in the context of high resolution nested models, problems may arise in the representation of spatial scales longer than the domain of the model. In this paper we study how well 4D-Var is able to estimate the whole range of spatial scales present in nested models. Using a model of the one-dimensional advection-diffusion equation we show that small spatial scales that are observed can be captured by a 4D-Var assimilation, but that information in the larger scales may be degraded. We propose a modification to 4D-Var which allows a better representation of these larger scales.

Keywords: advection-diffusion equation, sine transform, 4D-Var

1. Introduction

In many applications of environmental forecasting, such as numerical weather prediction, it is necessary to estimate the current state of the system in order to make a forecast. Usually the number of available measurements of the system is not sufficient to define the state uniquely and so the measurements are combined with a numerical model forecast, using techniques of data assimilation, in order to provide the best estimate of the system state. In operational numerical weather prediction a common data assimilation technique is that of four-dimensional variational data assimilation

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(4D-Var). This technique formulates the assimilation problem as an optimization problem over space and time, constrained by a numerical model of the equations describing the atmospheric flow. The solution to this optimization problem provides the state estimate from which a forecast can be produced.

An important challenge in weather prediction is the improvement of our ability to forecast small-scale localized weather systems, such as convective storms. Such systems are often associated with severe weather events, such to capture better the long-wave information by using our knowledge of how such waves are aliased. Finally we draw conclusions in section 4.

2. F[illegible]

2.1. Formulation

The aim of four-dimensional data assimilation (4D-Var) is to estimate the system state

is commonly used in data assimilation for nested models, for example at the Met Office [11], and it is the transform we will use in this paper. In this case the variables correspond to the different wave numbers in the discrete sine transform.

One possible problem with the use of this transform is the treatment of waves that have a wavelength longer than the domain of the nested model. In this case the true scale of the wave cannot be represented by the sine transform and so information from such long waves will be projected onto shorter scales by the transform. This is essentially a reverse of the classical aliasing problem. Whereas usually aliasing is considered as the misinterpretation of small-scale waves as larger-scale waves, here we have large-scale waves being misinterpreted as shorter-scale waves. To illustrate this effect we define a sine wave with wavenumber one over the periodic domain $[0, 1)$ and calculate its sine transform over the whole domain and then the sine transform of the part of the wave in the nested domain $[0, 0.25]$. The results are shown in Figure 1. Both transforms are calculated using 32 spatial points. We see that for the transform over the whole domain the power is all at wavenumber two. This is a property of the sine transform, in which the power of a sine wave of wavenumber k appears at wavenumber $2k$ when the transform is applied on a domain of length one. When the transform is applied on the smaller domain there is only a quarter wavelength that fits into the domain, which is not within the discrete spectrum on the nested model grid. In this case we see that most of the power is projected onto wavenumber one, with significant power also in higher wavenumbers.

Previous studies have examined different methods for treating the large spatial scales in a nested model data assimilation using a three-dimensional variational data assimilation scheme, a variation of 4D-Var in which the observations are all considered to be at the same time as the background. Within this context the authors of [1] examined the possibility of taking the large scales completely from a parent model analysis and using the nested model data assimilation to update only the small scales. An alternative method that has been proposed is to constrain the large scales to be close to those of the parent model analysis and a nested model background field by the addition of an extra term in the objective funtion [7]. Here we examine how the aliasing of the long waves affects their representation in a 4D-Var scheme and propose a new modification to the data assimilation system to allow for this.

[redacted]

$$u \quad + cu \quad = \quad u \quad ; \qquad (6)$$

where $u(x;t)$ is the temperature, x is the spatial coordinate, t is the time, $c \geq 0$ is the constant advection velocity, ≥ 0 is the di usion constant and subscripts indicate derivatives. The equation for the parent model is defined on the domain $x \in (0;1]$ with periodic boundary conditions.

The model is discretized using an explicit Euler scheme for the time derivative, centred di erences for the di usion term and upwind di erences for the advection term. We define a spatial step Δx and time step Δt. Then $u(x \ ; t \ )$ is approximated by $u_{\ ,\ }$, where for each point we have the discrete update equation

$$u_{\ , \ +1} = (\quad + \quad) u_{\ -1, \ } + (1 - \quad - 2 \quad) u_{\ , \ } + \quad u_{\ +1, \ } ; \qquad (7)$$

with $= c\Delta t = \Delta x$ and $= \Delta t = (\Delta x)^2$. The scheme is first-order in time and space, but is close to second-order spatial accuracy when the discrete Peclet number $c\Delta x =$ is small [6, p. 138].

For the nested model we discretize (6) on the limited domain $[0{:}5; 1]$, with the boundary conditions $u(0{:}5; t)$ and $u(1; t)$ taken from a run of the parent model at all times t. In the interior of the domain the discretization scheme is exactly as in the parent model, with a higher resolution spatial step Δx and time step Δt. Close to the boundaries the nested model solution is relaxed to the parent model solution using a Davies relaxation scheme over a small bu er zone [5].

3.2. Experimental design

Idealised 4D-Var experiments are set up by running the model from a known initial state, which we refer to as the true trajectory, and then generating observations from this true trajectory to use in the assimilation. The truth is generated at a higher resolution than either the parent or nested models, with spatial step Δx and time step Δt. For the experiments presented here we use values 5TfTf10.491.8Td[(and)351(e8re)4.8uTJ/F1Tf-332(ru2J/F1T849(e66.6

perfect analysis on the parent model grid which includes the components of the truth u at the initial time that can be represented on this grid, that is

$$u \ (x;0) = 5\sin \ x + \sin 2 \ x: \tag{9}$$

To generate a background field with known covariance for the nested model, we choose to add random noise to this field in spectral space at the nested model resolution. We interpolate u to the nested error covariance matrix Σ in (5) is defined to be a diagonal matrix, in which the first seven components are set to the true variance of the background error, 0.25. For the higher wavenumbers there is no useful information coming from the background, so we assume a variance of 5.0, which ensures that the observations will be given a much greater weight than the background at these scales. In Figure 2 we show the power spectrum of the error in the background and the error in the analysis. For clarity we show only the lowest and highest wavenumbers, which are the parts of the spectrum containing the true solution.

The first thing that we notice is that the data assimilation in the nested model is able to capture the high resolution information at wavenumber $k = 18$. This wave appears in the true solution, but it cannot be resolved by the parent model and so is set to zero in the background field. The use of the 4D-Var system with the high resolution nested model enables information at this scale to be inferred from the observations. Further experiments show that a necessary condition to infer this wave is that the observations are also at the high resolution; it is not su cient to have a high resolution data assimilation system with only low resolution observations [2]. At the low wavenumbers we see that the errors in the analysis are worse than those in the background for $k = 1$ and 2. The large-scale information coming from the background has been degraded during the assimilation process. Since these wavenumbers include information from the long wave that cannot be represented on the nested model domain, the nested assimilation does not treat this information correctly in this case.

In general we may expect the large scales provided by the parent model to be reasonably accurate and we would like to use the nested model assimilation to improve the small scales. Hence we would ideally like the 4D-Var scheme applied to the nested model to retain the large-scale information from the parent model. Since we have seen that this large-scale information is projected onto low wavenumbers by the sine transform, we may expect to improve the analysis if we constrain the solution to be closer to the background in these low wavenumbers. To test this possibility we run the assimilation experiment again, but within the matrix Σ we assume that variance on the low wavenumber components (k theerariance

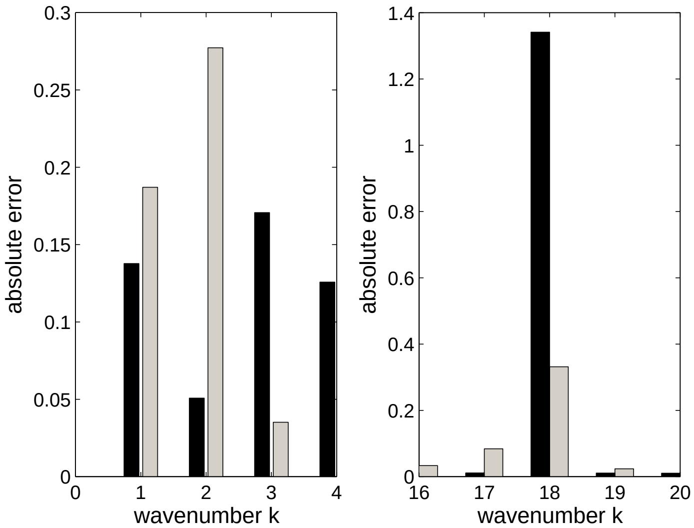

Figure 2: Power spectrum of errors in background (black) and analysis (grey) for low wavenumbers (left) and high wavenumbers (right) when the true error variances are used.

been much reduced with respect to the experiment using the true variances. The wavenumber one component of the solution is now more accurate than the background. At wavenumber $k = 2$ the analysis is still worse than the background, but it is improved with respect to the first experiment. Other choices of the variances at the low wavenumbers lead to further improvements at these scales [2]. An important aspect of this experiment is that the analysis at wavenumber $k = 18$ is still as accurate as in the first experiment. Hence, by over-weighting the low wavenumbers in the background field, we have been able to retrieve the small-scale information while retaining the accuracy of the background in the large scales.

4. C

The development of data assimilation schemes for very high resolution nested models is an important component of the development of future weather prediction systems. In this paper we have analysed one particular aspect of such schemes, namely the treatment of very long waves within a 4D-Var data assimilation system. Information on large spatial scales is provided to a nested model by a larger-domain parent model and it is important

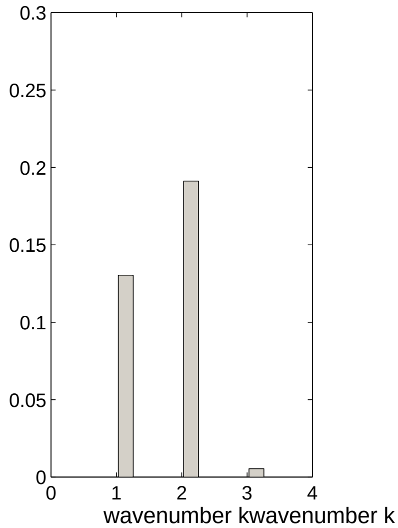

Figure 3: As Figure 2, for the experiment in which the low wavenumbers are over-weighted in the assimilation.

that the assimilation in the nested model does not degrade this information. We have shown that within a nested model domain these scales are projected onto low wavenumbers by a spectral transform. Hence the low wavenumbers in the background contain a combination of information at scales which the nested model can resolve and information at scales larger than the domain. When a standard 4D-Var assimilation is performed in spectral space it is not able to disinguish between these two sources of information. Hence the high resolution assimilation is able to improve the estimate of the state at small scales, but this occurs at the expense of a loss of information at the large scales. We have proposed a modification to 4D-Var for these cases, in which the low wavenumbers in the background are given more weight in order to allow for the fact that they contain information on larger scales than can be represented in the nested model. By performing the assimilation in spectral space and over-weighting the low wave numbers, we are able to improve the estimates of these large scales, while still keeping the same accuracy in the smaller scales.

Previous studies reported in [1] and [7] have also used the large scales of the parent model analysis as a constraint in the nested model analysis, in proach of [1] enforces the large scales of the nested model to be exactly equal to those of the parent analysis, that of [7] weakly constrains the large scales by a combination of these scales from the parent analysis and a previous nested model forecast. In the new approach presented here we use the large scales from only the parent analysis as a constraint, as in the work of [1], but they act as only a weak constraint and observations are allowed to alter

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