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A note on the analysis error associated with 3D-FGAT

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Abstract: The analysis error variance of a 3D-FGAT assimilation is examined analytically using a simple scalar equation. It is shown that the analysis error variance may be greater than the error variances of the inputs. The results are illustrated numerically with a scalar example and a shallow-water model.

KEY WORDS First guess at appropriate time; variational data assimilation

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1 Introduction

Data assimilation is widely used in weather and ocean forecasting to provide initial conditions for numerical forecast models. By combining observational data with an a priori, or background, estimate of the model state, it is possible to obtain an improved estimate of the current state of the system, known as the analysis. Many data assimilation techniques are based on Bayes' rule, which in the case of Gaussian errors is equivalent to a least squares fitting. For such techniques, provided that the errors in the observations and background state are correctly represented, the analysis obtained will be at least as accurate as the most accurate piece of input information, in a statistical sense. Thus the addition of more information into the assimilation procedure cannot degrade the analysis.

In practice many approximations must be made in designing data assimilation schemes for practical use. One such approximation is known as 3D-FGAT (first-guess at appropriate time), which can be considered as a half-way step between incremental three-dimensional variational data assimilation (3D-Var) and incremental aaure cannot

3 Analysis error for a simple example

term measures how close the identity approximation used in 3D-FGAT is to the true tangent linear model. The further the tangent linear model is from the identity, the larger this term will become. It is particularly important to note that for values of far from unity this term may be arbitrarily large and so the analysis error variance at the initial time may exceed the error variance of the inputs.

In this example we have assumed that the variance information of the background field is correct at the centre of the time window. However, by removing the model evolution of the perturbation, the evolution of the variance information is neglected, so that the innovations are weighted incorrectly. For the case where > 1 so that the variance grows throughout the assimilation window, then the innovation at time t_0 is over-weighted with respect to the background and the innovation at time t_2 is under-weighted (with the opposite occurring for < 1). It is this incorrect use of the statistical information which leads to a sub-optimal analysis.

If we consider 3D-Var scheme applied to this system, so that the observations are assumed to be valid at the centre of the time window, then the analysis is found to be no longer unbiased. This arises from the fact that the innovations are calculated as

$$d_0 = y_0 \quad x_b(t_0); \quad d_2 = y_2 \quad x_b(t_0). \tag{16}$$

This introduces terms in the expected analysis error dependent on the change in the true state between observation times. Terms involving the true state then also occur in the expression for the variance. Hence we see that 3D-FGAT theoretically removes a major source of error in 3D-Var, even if the analysis error variance may be large.

4 Numerical results

In order to illustrate the problems associated with the analysis error for 3D-FGAT we present numerical results based on the example presented in the previous section and on a shallow-water model. In practice we may expect the effect of the 3D-FGAT approximation to depend on the ratio of ${}^{2}_{b} = {}^{2}_{o}$

4.2 Shallow water model

As a second example we consider a more realistic system, the one-dimensional nonlinear shallow water system for the flow of a fluid over an obstacle in the absence of rotation. The model equations are given by

$$\frac{Du}{Dt} + \frac{\partial}{\partial x} = \quad g\frac{\partial h}{\partial x}; \qquad \frac{1}{}\frac{D}{Dt} + \frac{\partial u}{\partial x} = 0; \tag{17}$$

with $D/Dt = \partial/\partial t + u\partial/\partial x$. In these equations $h = h(x)$ is the height of the bottom orography, u is the velocity of the fluid and $= gh$ is the geopotential, where g is the gravitational acceleration and $h > 0$ the depth of the fluid above the orography. The system is discretized using a semi-implicit semi-Lagrangian integration scheme, as described in Lawless *et al.* (2003). We define the problem on a periodic domain of 1000 grid points, with a spacing $x = 0.01\ m$ between them, so that $x \in [0\ m; 10\ m]$ and assume a model time step $t = 0.0092\ s$. Other parameters for the problem are as defined in Lawless *et al.* (2005).

A 3D-FGAT scheme for this system is set up over a time window $[\ T; T]$, with observations of u and at every spatial grid point. T0.0097at16.1592(y).0092(>)u67(T)]Tr5yTJ/F.285 0 Td .285 0 T .28 Td [(dpmodel)-298(time)r 0 T .odets, with3D-ndepthf292(hf)econdi37(of)]Tf60is).

this component of the error within the assimilation system may lead to a sub-optimal analysis, which can cause an increase in the analysis error even when all other prior statistical information is correctly specified. It is important not to confuse this assumption with the tangent linear assumption used in 4D-Var. Even in a linear situation the assumption of 3D-FGAT may not hold.

Although these results have been demonstrated for simple examples, there is no reason to think that this problem will disappear as the model becomes more complicated. Rather, the problem may arise whenever the true tangent linear model matrix is far from the identity. However, it must be recognized that 3D-FGAT is still likmode58osill i airiis m.