University of Reading

Pe e pence

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Abstract



Declaration



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Chapter 1

Introduction

1.1 Neutron Tube Basic Operation



Neutron tu e sche atic



1.2 Free Boundary Problem





1.3 Solution Approach

2.1 Analytic Solution

n (x) (x $\rho(x) = \frac{J}{v(x)},$ $p(x) = \frac{J}{v(x)},$ p(x) = $F = qE = -q\frac{dU}{dx}$ $= mv(x)\frac{dv}{dx}$ 🔺 n 🗸 a $\frac{m}{2}\frac{d}{dx}v(x)^2 = -q\frac{dU}{dx}$ a _____i ia^a in a^ain i $v(x)^2 = -\frac{2q}{m}U(x) + c_1$ $\begin{array}{c} \mathbf{\dot{x}} w & c_1 & \text{in} & \mathbf{\dot{c}} & \mathbf{\dot{n}} & \mathbf{$ $c_1 = v_0^2 + \frac{2q}{m}U_1$ **a**n $v(x) = \frac{s}{\frac{2q}{m}(U_1 - U(x)) + v_0^2}$



CHAPTER 2. 1D PLANAR CASE





 \leftarrow Change in solution region size with e \rightarrow ission current density

2.2.2 Variation in Solution Region Size with Accelerating Potential

A^ain^a i y nin ti s t s ?

nein
$$U(x)$$
 in \mathcal{W} is n^a of n n nein $\phi(x,\tau)$. An
a of ni a a a a $\tau \to \infty$ nein $\phi(x,\tau)$ c n of
y of n $U(x)$ i. a $\tau \to \infty$ $\phi_{\tau} \to 0$ c of a final
a final fina

2.5 Mapping from Physical Grid to Logical Grid

$$\frac{\partial c}{\partial \tau} = \frac{1}{2} \sum_{x_{1-1}}^{x_{1}} \frac{\partial M}{\partial \tau} dx - \frac{1}{2} \sum_{x_{1}}^{x_{1}} \frac{\partial M}{\partial \tau} dx - \frac{1}{2}$$

CHAPTER 2. 1D PLANAR CASE

 $\frac{\partial}{\partial \tau} \int_{x_0}^{x_n} \phi dx$

2.6.2 Gradient Dependent Monitor Function

$$M = 1 + \gamma \phi_x \gamma \text{ in } a \text{ cn} a \text{ n } n$$

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$$\phi_i - \phi_{i-1}$$
 inc^a inc^a x_i

CHAPTER 2. 1D PLANAR CASE

$$\mathbf{A} = \overset{\mathbf{O}}{\overset{\mathbf{B}}{\underline{R}}} \overset{\mathbf{1}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}{\underline{R}} \overset{\mathbf{O}}{\underline{R}}} \overset{\mathbf{O}}$$

CHAPTER 2. 1D PLANAR CASE





2.9.1 Boundary Quadratic Velocity

CHAPTER 2. 1D PLANAR CASE



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CHAPTER 2. 1D PLANAR CASE

Chapter 3

1D Radial Case

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y c i n a i n i n a i a
i a n c n c i n c i n a i a
i a n c i n a i a
i a n c i n a i a
a n i n a i a

$$\nabla^2 U(x,y) \rightarrow \frac{1}{r^2} \frac{\partial^2 \psi(r,\theta)}{\partial \theta^2} + \frac{\partial^2 \psi(r,\theta)}{\partial r^2} + \frac{1}{r} \frac{\partial \psi(r,\theta)}{\partial r} = -\frac{\rho(r,\theta)}{\epsilon_0}$$
i i n a i a n i a n i a n i a i a c i n a $\psi(r,\theta)$.

3.1 Problem Construction

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3.2 Mapping from Physical Grid to Logical Grid

ince en aychin in a in n y w c in a an in n c y in n in yic² in n ic² in A in cn i i i i n n in ic² in c n in i a n di in yic² in

$$\lim_{n \to \infty} \frac{2}{\sigma} = \frac{\xi}{\xi_n(\tau) - \xi_0} \operatorname{cn}^{\sigma} \frac{2}{n} \operatorname{in}^{\sigma} = \frac{1}{\frac{1}{(\tau)} \sum_{r_{i-1}}^{r_i} Mr dr} = c_{i-\frac{1}{2}}$$

$$\frac{1}{2}$$
 $\frac{1}{2}$ $\frac{1}$

$$\frac{\partial c}{\partial r}\frac{dr}{d\tau} = \frac{1}{(\tau)} Mrr_{r_{i-1}}^{r_i}$$

$$\vec{\Delta r} c = \frac{1}{2} \sum_{r_{i-1}}^{z} M \underline{r} dr + \frac{1}{2} \sum_{r_{i-1}}^{z} \frac{\partial M}{\partial \tau} r dr - \frac{1}{2} \sum_{r_{i-1}}^{z} M r dr$$

$$-\frac{i n n}{z} - \frac{a}{r_{i}} - \frac{z}{r_{i}} - \frac{z}{r_{i-1}} \frac{\partial M}{\partial \tau} r dr$$

$$-\frac{z}{r_{i-1}} - \frac{z}{r_{i-1}} Mr dr + Mr r \frac{r_{i}}{r_{i-1}} = 0 - 1$$

3.3 Choice of Monitor Function

$$M = \mathbf{1} + \frac{\gamma}{r} \phi_r \cdot \mathbf{n}$$

$$\frac{dr}{d\xi} = \frac{1}{r + \gamma \phi_r},$$

 i_n in nein^an ^an ^ai $\xi/(\xi_n(\tau) - \xi_0)$ i ny

$$\frac{\xi}{\xi_n(\tau) - \xi_0} = \frac{\frac{1}{2} r_i^2 - r_{i-1}^2 + \gamma (\phi_i - \phi_{i-1})}{\frac{1}{2} (r_n^2 - r_0^2) + \gamma (\phi_n - \phi_0)}$$
$$= c_{i-\frac{1}{2}}$$

in $n^2 y^2 r_n(\tau)$ in n^2 is right n in n^2 or n^2 in n^2 is right right of n^2 or n^2 in n^2 in n^2 is n^2 in n^2





$$n^{\sigma} = n^{\sigma} \phi_{n} = 0^{\alpha} n \quad I_{3} \quad y$$

$$\sum_{r_{0}}^{r_{n}} \phi_{r} dr \approx \frac{1}{2} \sum_{i=1}^{\infty} (r_{i} - r_{i-1})(\phi_{i} r_{i} + \phi_{i-1} r_{i-1})$$

$$= \frac{1}{2} (r_{n} - r_{n-1}) \phi_{n} r_{n} + \frac{1}{2} \sum_{i=1}^{\infty} (r_{i+1} - r_{i-1}) \phi_{i} r_{i} \qquad (\mathbf{v} = \mathbf{v} + \mathbf{v}$$

$$\sum_{j=i+1}^{n} c_{j-\frac{1}{2}} = \frac{1}{2} (r_n^2 - r_i^2) + \gamma (\phi_n - \phi_i)$$

$$\phi_i = \phi_n - \frac{1}{\gamma}\overline{C_i} + \frac{1}{2\gamma}(r_n^2 - r_i^2)$$

$$\mathbf{P}_{i}^{n} \quad \mathbf{P}_{j=i+1}^{n} c_{j-\frac{1}{2}} = \overline{C_{i}}^{n} \mathbf{a}$$

$$\mathbf{T}^{a} \mathbf{i}_{n} \mathbf{\sigma} \mathbf{\sigma} \mathbf{i} \quad \mathbf{i}^{a} \mathbf{i}_{1}$$

$$\phi_i = -\frac{-}{\gamma}\overline{C_i} + \frac{1}{\gamma}(r_n\underline{r}_n - r_i\underline{r}_i)$$

$$\frac{1}{2} \sum_{i=1}^{\infty} \phi_i - \frac{r_i^2}{\gamma} (r_{i+1} - r_{i-1}) \underline{r}_i + \frac{1}{2} (3r_n - r_{n-1}) \phi_n + \cdots$$

$$\cdots + \frac{r_n}{\gamma} \sum_{i=1}^{\infty} r_i (r_{i+1} - r_{i-1}) \underline{r}_n - \frac{1}{2\gamma} \sum_{i=1}^{\infty} r_i (r_{i+1} - r_{i-1}) \overline{C_i}$$

$$= 0$$

3.5 Results and Conclusions From







Chapter 4

Conclusions and Further Work

4.1 Conclusion

inine eine a a Υ . 'n J. ⊿wn n k n in aa_n aa \mathbf{a}_{n} а e ic c n a a a vica y ini i y ĺ n' . ¢in ₹ а , n , * а iin^a **n** n a . ٦. у $nc \perp n$ c n ↓ n ã C an c, n niy 🦾 C C а cc ↓n 🤨 n 🕯 n 🏄 , C c n n à а y∔c^a ⊥n c n " n n y inc n a cin " inc^a a a а ini n n у 'n 1 a a a n y y in 🔬 \downarrow n 'n c ic cc а а C. dn i in i . A in " n តុរ្ភ i n ð, d n y i . e ic 'n c n n а a a а а , P 'n į, i i n nc Ti . . n. V а . n y. n **a** , a a a cc ain , a iin.⊿wcn , C ↓n а ein ini in 🛓 $\dot{\mathbf{n}}$. . aa n y in g and a in T а y∤ n 1 a a wy y c eic T in 🤊 i v i n h n c n

CHAPTER 4. CONCLUSIONS AND FURTHER WORK

CHAO 112.4.CONCLUSIONSANDFU TH. WORK

Acknowledgements

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Bibliography

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- ▼ j n $\stackrel{\mathbf{a}}{=}$ n. Fee n vng vn P le $\stackrel{\mathbf{a}}{=}$ n n ¬ ▼
- jMg ^an⊿wint_i → ^an n a v n