University of Reading

The Position of the Free Boundary Formed Between an Expanding Plasma and an Electric Field in Different Geometries

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[illegible]

Abstract

[illegible]

Declaration

[illegible]

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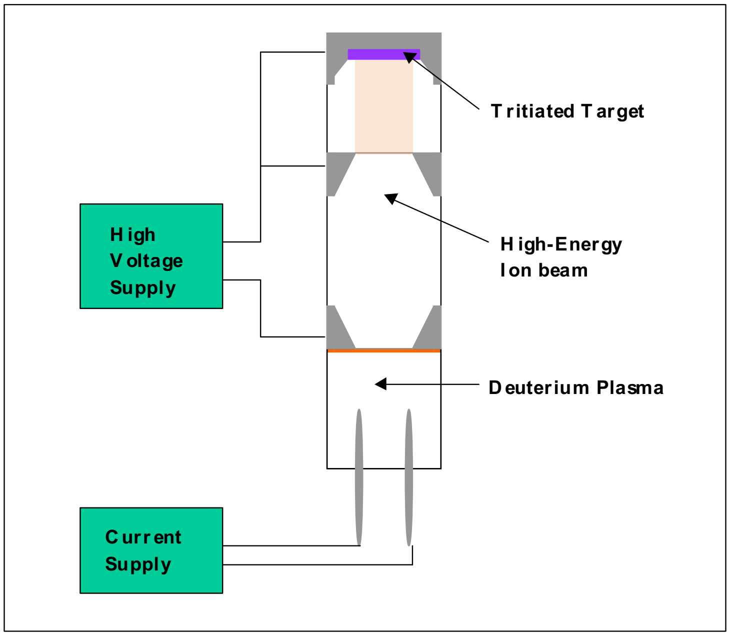
Chapter 1

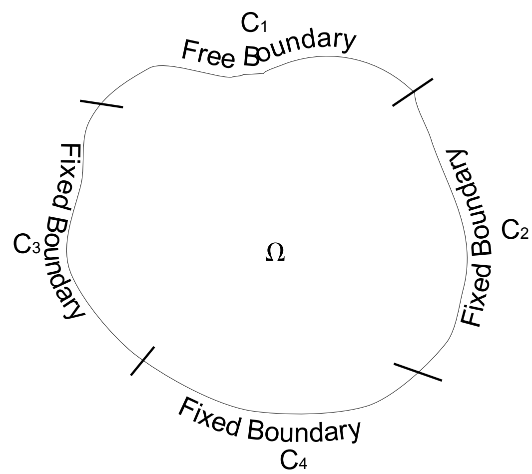
Introduction

1.1 Neutron Tube Basic Operation



Neutron tube schematic

1.2 Free Boundary Problem



Sche atic of D free oundary pro le

C_1

C_4 n C_1

$$\frac{\partial U}{\partial n} = 0$$

C_1

$$\nabla^2 U = \frac{-\rho(x,y)}{\epsilon_0}$$

[illegible]

[illegible]

1.3 Solution Approach

[illegible]

[illegible]

[illegible]

[illegible]

2.1 Analytic Solution

$$\rho(x) = \frac{J}{v(x)},$$

$$\begin{aligned} F &= qE = -q\frac{dU}{dx} \\ &= mv(x)\frac{dv}{dx} \end{aligned}$$

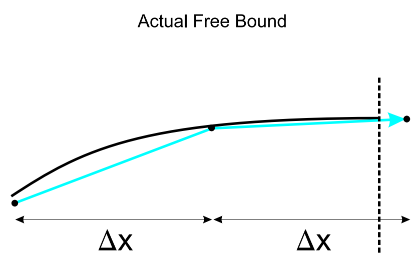
$$\frac{m}{2}\frac{d}{dx}\, v(x)^2 = -q\frac{dU}{dx}$$

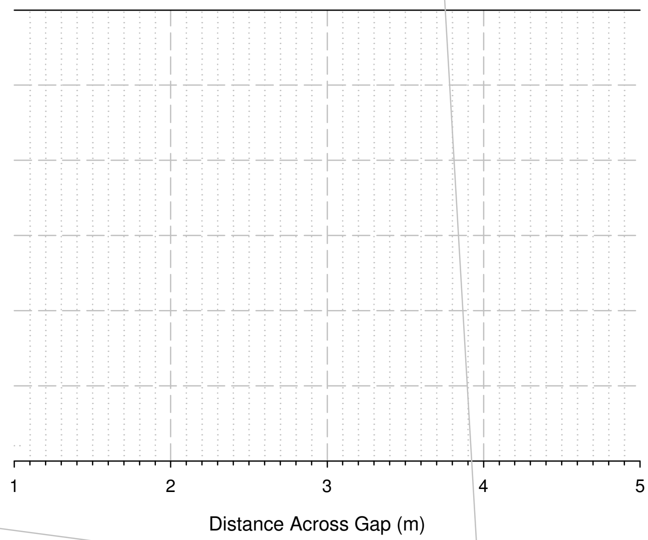
$$v(x)^2 = -\frac{2q}{m}U(x) + c_1$$

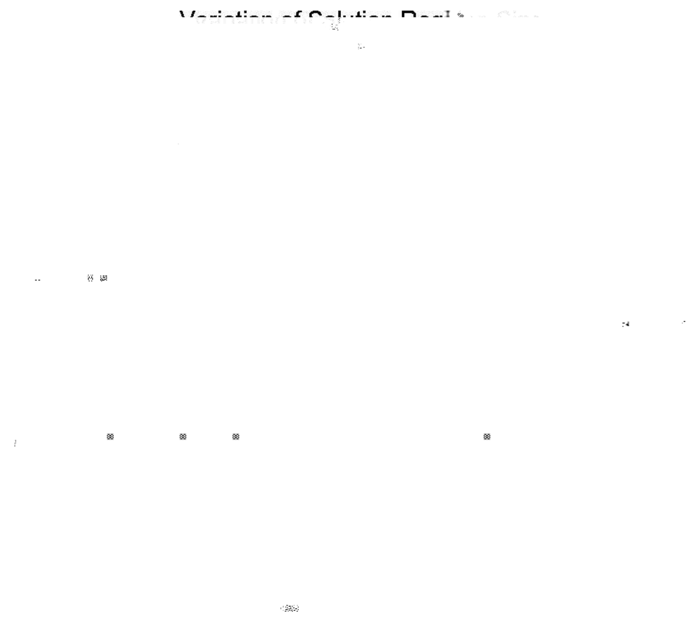
$x = S$ $U = U_1$ $v(x) = v_0$

$$c_1 = {v_0}^2 + \frac{2q}{m}U_1$$

$$v(x) = \sqrt{\frac{2q}{m}\left(U_1 - U(x)\right) + {v_0}^2}$$


Actual Free Bound
Δx
Δx


1
2
3
4
5
Distance Across Gap (m)



Change in solution region size with e ission current density

2.2.2 Variation in Solution Region Size with Accelerating Potential


Distance From Fixed Boundary (m)

nc n $U(x)$ n n n nc n $\phi(x,\tau)$. An n $\tau \to \infty$ nc n $\phi(x,\tau)$ c n y n $U(x)$ $\tau \to \infty$ $\phi_\tau \to 0$ c n n

2.5 Mapping from Physical Grid to Logical Grid

T n c n yc n n n n n y nc n n τ. n c c c y n n c n yc n n nc n n n n y n y n c n n $\phi(x,\tau)$ n c c n n n n n y nc.

M n n n y c n n c n n n n c n c n n c n n c n n n n T n T y T

$$\frac{\partial c}{\partial \tau} = \frac{1}{\ } \int_{x_{i-1}}^{x_i} \frac{\partial M}{\partial \tau} dx - \frac{-}{2} \int^{x_i}$$

[illegible] x_i [illegible] x_{i-1} [illegible] i^{th} [illegible] x_0 [illegible] x_1 [illegible]

[illegible] $\underline{x}$ [illegible]

$$[\underline{x}]_{x_{i-1}}^{x_i} - \int_{x_{i-1}}^{x_i} dx = 0,$$

[illegible] i^{th} [illegible]

$$\underline{x}_i - x_{i-1} - c_{i-\frac{1}{2}} = 0$$

[illegible]

$$\frac{d}{d\tau}\int_{x_0}^{x_n} \phi dx = \frac{\partial}{\partial\tau}\int_{x_0}^{x_n} \phi dx + [\phi \underline{x}]_{x_0}^{x_n}$$

[illegible]

$$\frac{\partial}{\partial\tau}\int_{x_0}^{x_n} \phi dx$$

$$\frac{1}{2}\sum_{i=1}^{n-1}(\phi_{i-1}-\phi_{i+1})\underline{x}_i + \frac{1}{2}(3\phi_n+\phi_{n-1})\underline{x}_n + \frac{1}{2}\sum_{i=1}^{n-1}(x_{i+1}-x_{i-1})\underline{\phi}_i = $$
$$-\left.\frac{\partial\phi}{\partial x}\right|_{x_0} + \frac{1}{2}\sum_{i=1}^{n-1}(x_{i+1}-x_{i-1})g(\phi_i) + \cdots$$
$$\cdots + \frac{(x_1-x_0)}{2}g(\phi_0) + \frac{(x_n-x_{n-1})}{2}g(\phi_n)$$

$M = 1$ ... $\underline{x}$

2.6.2 Gradient Dependent Monitor Function

$M = 1 + \gamma\phi_x$ ... γ ... ϕ_x ... $\xi/(\xi_n(\tau)-\xi_0)$

$\phi_i - \phi_{i-1}$ nc nc x_i

ϕ_i … $c_{i-\frac{1}{2}}$ … $i+1$ … n

$$\sum_{j=i+1} c_{j-\frac{1}{2}} = \frac{\gamma(\phi_n - \phi_i) + (x_n - x_i)}{}$$

ϕ_i

$$\phi_i = \phi_n - \frac{}{\gamma}\overline{C_i} + \frac{1}{\gamma}(x_n - x_i)$$

$\overline{C_i}$

$$\phi_i = -\frac{-}{\gamma}\overline{C_i} + \frac{1}{\gamma}(x_n - x_i),$$

x

$$\frac{1}{2}\sum_{i=1}^{n-1}\left((\phi_{i-1} - \phi_{i+1}) - \frac{1}{\gamma}(x_{i+1} - x_{i-1})\right) x_i + \frac{1}{2}(3\phi_n + \phi_{n-1}) + \cdots$$

$$\cdots + \frac{1}{\gamma}\sum_{i=1}^{n-1}(x_{i+1} - x_{i-1})\Bigg)\, x_n - \frac{1}{2\gamma}\sum_{i=1}^{n-1}\overline{C_i}(x_{i+1} - x_{i-1})\Bigg) -$$

$$= \frac{\partial \phi}{\partial x}_{x_0} + \frac{1}{2}\sum_{i=1}^{n-1}(x_{i+1} - x_{i-1})g(\phi_i) + (x_1 - x$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{B} & \underline{c}_1 \\ \underline{R} & s \end{pmatrix}, \qquad \underline{b} = \begin{pmatrix} \underline{b}_1 \\ e \end{pmatrix}.$$

$\mathbf{B}$ … $n \times n$ … n …

$\underline{c}$ … n … $n + 1$ …

$\underline{R}$ … n …

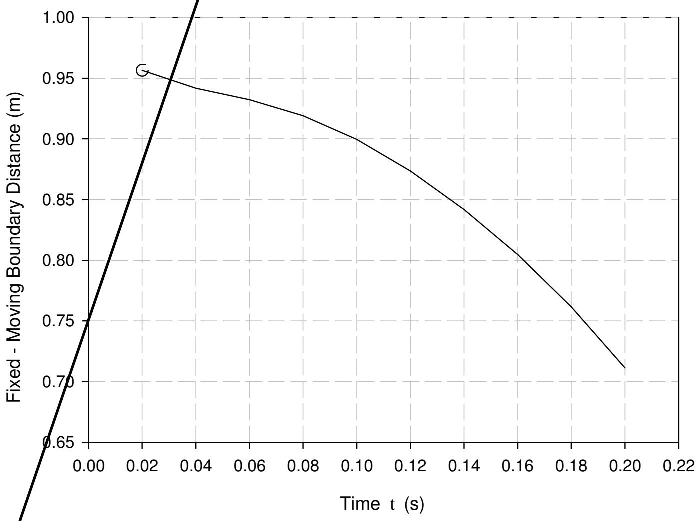
M y n

$$\mathbf{D}\underline{x} + \underline{c_2} - = \underline{b_2}$$

c

$$\underline{x} = \mathbf{D}$$

2.9.1 Boundary Quadratic Velocity



1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16 0.18 0.20 0.22
Fixed - Moving Boundary Distance (m)
Time t (s)

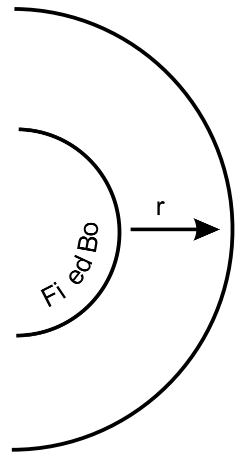
Chapter 3

1D Radial Case

$$\nabla^2 U(x,y) \rightarrow \frac{1}{r^2}\frac{\partial^2\psi(r,\theta)}{\partial\theta^2} + \frac{\partial^2\psi(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial\psi(r,\theta)}{\partial r} = -\frac{\rho(r,\theta)}{\epsilon_0}$$

$U(x,y)$ ... $\psi(r,\theta)$.

3.1 Problem Construction


r
Fi ed Bo

$$\frac{\partial \phi}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi(r)}{\partial r}\right) + g(\phi)$$

3.2 Mapping from Physical Grid to Logical Grid

$r_n(\tau)$ c n in n y S n

A n n n n y c n

$$\int_{r_{i-1}}^{r_i} Mrdr = \int_{\xi_{i-1}}^{\xi_i} d\xi$$
$$= \xi$$

n $\xi/(\xi_n(\tau)-\xi_0)$ c n n n

$$\frac{1}{(\tau)}\int_{r_{i-1}}^{r_i} Mrdr = c_{i-\frac{1}{2}}$$

n n n n $\underline{r}$ n n

n y n

$$\frac{dc}{d\tau} = 0 = \frac{\partial c}{\partial r}\frac{dr}{d\tau} + \frac{\partial c}{\partial \tau}$$

n n n y

$$\frac{\partial c}{\partial r}\frac{dr}{d\tau} = \frac{1}{(\tau)} \; Mr\underline{r} \Big|_{r_{i-1}}^{r_i}$$

c n n y n

$$\frac{\partial c}{\partial \tau} = \int_{r_{i-1}}^{r_i} \frac{\partial}{\partial \tau}\left(\frac{Mr}{(\tau)}\right) dr = 0$$

c n n n

$$\frac{\partial c}{\partial \tau} = \frac{1}{\;}\int_{r_{i-1}}^{r_i} M\underline{r}dr + \frac{1}{\;}\int_{r_{i-1}}^{r_i} \frac{\partial M}{\partial \tau} rdr - \frac{\underline{\;}}{\;^2}\int_{r_{i-1}}^{r_i} Mrdr$$

n n n

$$\int_{r_{i-1}}^{r_i} M\underline{r}dr + \int_{r_{i-1}}^{r_i} \frac{\partial M}{\partial \tau} rdr - \frac{\underline{\;}}{\;}\int_{r_{i-1}}^{r_i} Mrdr + \; Mr\underline{r}\Big|_{r_{i-1}}^{r_i} = 0$$

$(n+1)\times n$ n $\underline{r}$ $r_0 = 0$

$$\frac{d}{d\tau}\int_{r_0}^{r_n} \phi r dr = \int_{r_0}^{r_n} \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi(r)}{\partial r} \right) + g(\phi) \right) r dr$$

$\underline{r}$

3.3 Choice of Monitor Function

$M = 1$

$M = 1 + \frac{\gamma}{r}\phi_r$

$$\frac{dr}{d\xi} = \frac{1}{r+\gamma\phi_r},$$

ϕ_r

$\xi/(\xi_n(\tau)-\xi_0)$

$$\begin{aligned} \frac{\xi}{\xi_n(\tau)-\xi_0} &= \frac{\frac{1}{2}\left(r_i^2 - r_{i-1}^2\right) + \gamma\left(\phi_i - \phi_{i-1}\right)}{\frac{1}{2}\left(r_n^2 - r_0^2\right) + \gamma\left(\phi_n - \phi_0\right)} \\ &= c_{i-\frac{1}{2}} \end{aligned}$$

$r_n(\tau)$ r

i^{th}

nc $(\phi_i - \phi_{i-1})$ nc n nc

c n n i n n nc

n c n y n i n n n c

n $M = 1 + \frac{\gamma}{r}\phi_r$ n

$$\int_{r_{i-1}}^{r_i} \left(1 + \frac{\gamma}{r}\phi_r\right) r\,dr = \frac{\partial}{\partial \tau}\int_{r_{i-1}}^{r_i} r\,dr + \gamma \int_{r_{i-1}}^{r_i} \frac{r\phi_r}{r}\,dr$$

$$\int_{r_{i-1}}^{r_i} \frac{\partial M}{\partial \tau} r\,dr \quad \ldots \quad r\,dr$$

n

n n

n

$$\gamma \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi(r)}{\partial r}\right) + g(\phi)\right]_{r_{i-1}}^{r_i} + [r r]_{r_{i-1}}^{r_i} \ldots + \gamma\phi_r)\, r]_{r_{i-1}}^{r_i}$$

$\ldots$

c

$\gamma\phi_r)$

n I_1

$$\gamma \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi(r)}{\partial r}\right) + g(\phi) \right]_{r_{i-1}}^{r_i} = \gamma \left\{ \frac{1}{r_i}\frac{\partial}{\partial r}\left(r_i \left.\frac{\partial\phi}{\partial r}\right|_{r_i} \right)\right|_{r_i} - \cdots$$

$$\cdots - \frac{1}{r}$$

$\phi_n = 0$ I_3

$$\int_{r_0}^{r_n} \phi \underline{r}\, dr \approx \frac{1}{2}\sum_{i=1} (r_i - r_{i-1})(\phi_i \underline{r}_i + \phi_{i-1}\underline{r}_{i-1})$$
$$= \frac{1}{2}(r_n - r_{n-1})\phi_n \underline{r}_n + \frac{1}{2}\sum_{i=1}^{n-1}(r_{i+1} - r_{i-1})\phi_i \underline{r}_i$$

ϕ_i $\underline{r}$ $c_{i-\frac{1}{2}}$

$$\sum_{j=i+1} c_{j-\frac{1}{2}} = \frac{1}{2}(r_n^2 - r_i^2) + \gamma(\phi_n - \phi_i)$$

$$\phi_i = \phi_n - \frac{}{\gamma}\overline{C_i} + \frac{1}{2\gamma}(r_n^2 - r_i^2)$$

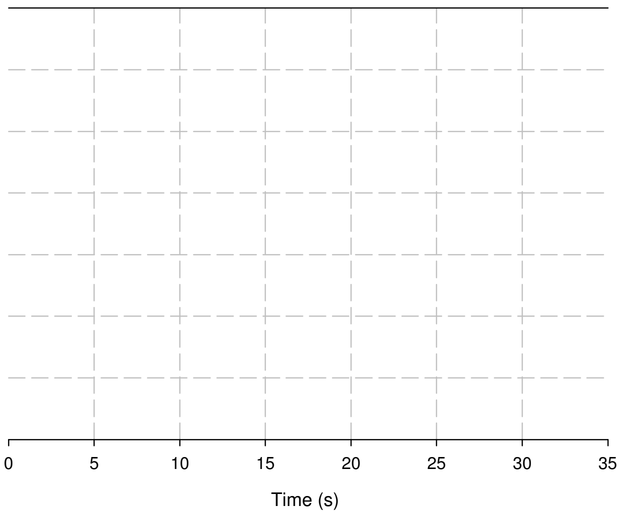
$\sum_{j=i+1}^{n} c_{j-\frac{1}{2}} = \overline{C_i}$

$$\phi_i = -\frac{}{\gamma}\overline{C_i} + \frac{1}{\gamma}(r_n \underline{r}_n - r_i \underline{r}_i)$$

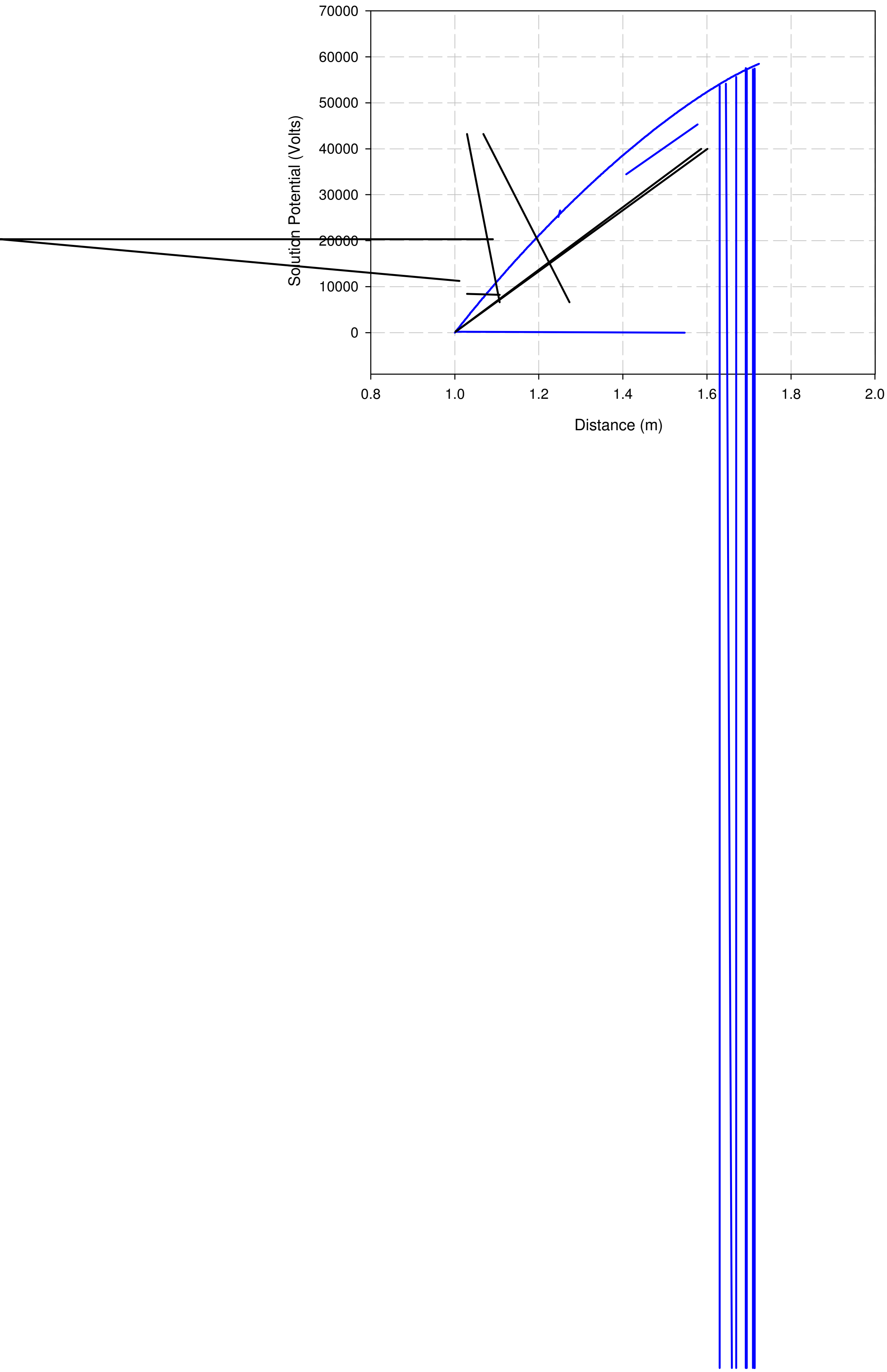
$\underline{r}_i$ $\underline{r}_n$

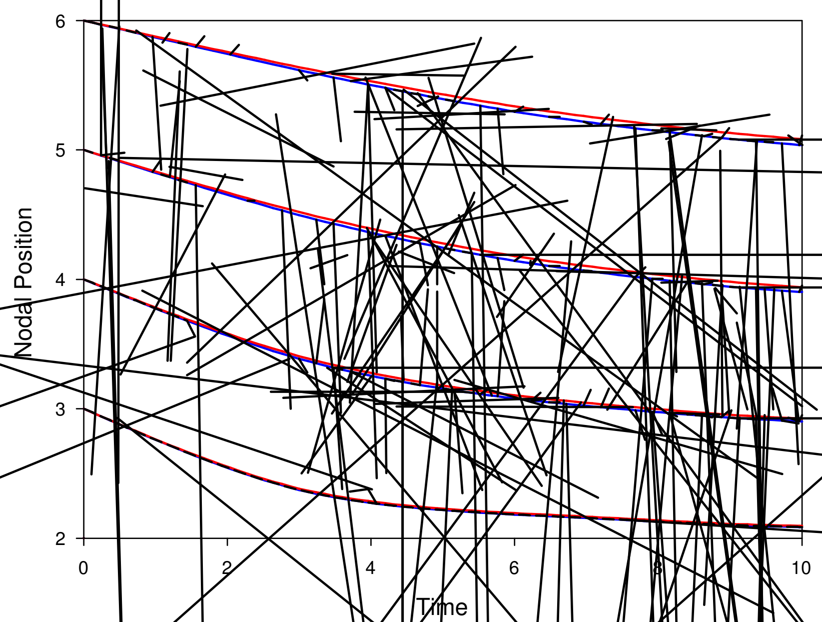
$$\frac{1}{2}\sum_{i=1}^{n-1}\left(\phi_i - \frac{r_i^2}{\gamma}\right)(r_{i+1} - r_{i-1})\underline{r}_i + \frac{1}{2}\left\{(3r_n - r_{n-1})\phi_n + \cdots\right.$$
$$\left.\cdots + \frac{r_n}{\gamma}\sum_{i=1}^{n-1} r_i(r_{i+1} - r_{i-1})\right\}\underline{r}_n - \frac{}{2\gamma}\sum_{i=1}^{n-1} r_i(r_{i+1} - r_{i-1})\overline{C_i}$$
$$= 0$$

3.5 Results and Conclusions From



0
5
10
15
20
25
30
35
Time (s)


70000
60000
50000
40000
30000
20000
10000
0
Solution Potential (Volts)
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Distance (m)


6
5
4
3
2
0
2
4
6
8
10
Nodal Position
Time

Chapter 4

Conclusions and Further Work

4.1 Conclusion

Acknowledgements

Bibliography

n n . *V cuu u e* Mc

M n *e l.* A n n n n n M n M A *P cee ng ell ng n S ul n n C e c l Eng nee ng C P ug l* A c n y .

M n . A L n n M n n n M nc n M n n *P cee ng SCA C n e ence H ng K ng* n .

n n . *F ee n v ng un P le* n n

Mc n T n . T n