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Variational data assimilation for parameter estimation: application to a simple morphodynamic model

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Variational data assimilation for parameter estimation: application to a simple morphodynamic model

Data assimilation is a technique for combining observational data with model predictions to 1) produce a model state that most accurately approximates the current and future states of the true system and 2) provide estimates of the model parameters. Whilst it is routinely used in atmospheric and oceanic prediction, the possibility of transferring data assimilation techniques to coastal morphodynamic modelling and prediction has only recently been investigated. In a precursor to the current work, Scott and Mason (2007) explored the use of data assimilation for state estimation in estuarine morphodynamic modelling using Morecambe Bay as a study site. A 2DH decoupled morphodynamic model of these parameters, where q is the number of unknown parameters. We assume that

3 The model

For the purpose of demonstrating the data assimilation technique we consider the simple case of a model with a single unknown parameter. For this we use the one-dimensional linear advection model described in Smith et al. (2007)

$$\frac{\partial z}{\partial t} + A\frac{\partial z}{\partial x} = 0, \tag{8}$$

where $z(x,t)$ is the bathymetry or bed height, A is the (constant) advection velocity and t is the time.

As discussed in Smith et al. (2007) we can use the method errors. Since, by the nature of the problem, these errors are not known exactly they have to be approximated in some manner.

Formulation of the background error covariance can be made considerably easier by specifying the error correlations as analytic functions. A number of correlation models have been proposed (see Daley (1991) for further discussion on this). An approach commonly used by the numerical weather prediction (NWP) community is the NMC method (Parrish and Derber (1992)) which uses the difference between forecasts that verify at the same time. The literature gives various other methods, including using innovation (observation minus background) statistics and studying differences in background fields using ensemble techniques. Fisher (2003) provides a useful review of current NWP techniques.

State covariance A standard approach used in state estimation is to assume that the background error covariances are homogeneous and isotropic. $\mathbf{B_{zz}}$ is then equal to the product of the estimated error variance and a correlation matrix defined using a pre-specified correlation function. Although this method is somewhat crude it makes the data assimilation problem far more tractable.

To characterise the background errors in the state vector $\mathbf{B_{zz}} = \{b_{ij}\}$ we use the correlation function (Rodgers (2000))

$$b_{ij} = \sigma_b^2 \rho^{|i-j|}, \qquad i,j = 1,\ldots,m. \tag{14}$$

Element b_{ij} defines the covariance between components i and j of the error vector $\boldsymbol{\varepsilon}_b$. Here $\rho = \exp(-\Delta x/L)$ where Δx is the model grid spacing and L is a correlation length scale and σ_b^2 is the state background error variance.

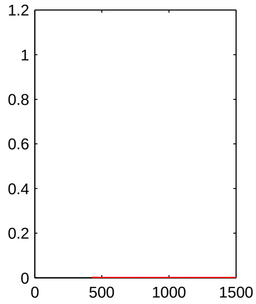
Parameter covariance For our simple model (8) we only have a single unknown parameter, the parameter vector $\mathbf{p}^b$ is therefore scalar. We approximate the true advection velocity A with $\tilde{A}$ where $\tilde{A} = A + \varepsilon_A$. Setting $\boldsymbol{\varepsilon}_p = \varepsilon_A$ in (13) we have

$\mathbf{B_{pp}} = E(\varepsilon$

about $\tilde{f}(x - At)$, yielding

$$\varepsilon_b(x,t) = \tilde{f}(x - At - \varepsilon_A t) - f(x - At)$$

$$= \tilde{f}($$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