University of

A Nyström Method for a Boundary

and

Peter G. Chamberlain[†] and Simon N. Chandler-Wilde[‡]

Department of Mathematics, University of Reading, PO Box 220,

Reading RG6 6AX, UK

September 3, 2009

Abstract

This paper is concerned with solving numerically the Dirichlet boundary value problem for Laplace's equation in a non-locally perturbed half-plane. This problem arises in the simulation of classical unsteady water

present and analyse a numerical scheme for computing the Dirichlet-to-Neumann map. i.e.

for $\mathbf{x_1}$ $\mathbb{R}$

ꝫ, t ꝫt **f** ꝫn **v** **v₁,v₂** ꝫt f

$$\frac{\partial \varphi}{\partial t} \quad -\frac{1}{2}\mathbf{v}^2 - g\mathbf{f},$$

$$\frac{\partial \mathbf{f}}{\partial t} \quad \mathbf{v}_2 - \mathbf{v}_1 \mathbf{f}'.$$

, for ꝫt on ꝫ o , pꝫrꝫt , t , t r nꝫt on of t , , o t pot nt ꝫ ꝫt ꝫn n t , fro t , o t on of t , t o pꝫrꝫ t r t , o n ꝫr po t on ꝫn t , D r t o n ꝫr ꝫtꝫ pꝫrꝫt on nꝫt rꝫ nꝫ , t , t , to , o , , , p t t , t pp n n r ꝫ t o , t ro o t t , ꝫt r ꝫ , t rꝫt r , for , ꝫ p , n , ttꝫ ꝫn A ꝫ Bꝫ fort , , , , Bꝫ r B ꝫ , B ꝫ , o Z ꝫn

t p , o t ꝫt t , n , ontr t on ꝫr , n t pꝫp r A ꝫ n no , t t ꝫt t pꝫp r ꝫpp ꝫr to , t , r t p ꝫt on to tꝫ , t , n r ꝫ o t on of t , o n ꝫr ꝫ , pro , n t , n r ꝫ ꝫ , of ꝫr trꝫr o n , ont n o D r t ꝫtꝫ φ_Γ t n , t r t , o n ꝫr nor φ_Γ ꝫΓ()09(n)28.0111((t)0.4362e)-414.647(pr)-0.39949(n)-427.285(t)90.15979.403(R0.11102e3749(b)089(r)d1159.6060.4320.512(I394541)0.44

1l(-)-0.43820.430062(i)-424.44@

for o p t n t , D r t to N , ɜnn ɜp to pro , ɜ too for t , ɜ n

o p tɜt onɜ pro , ɜt ɜ t , t p for pro , of ɜt on of non

p r o ɜt r ɜ , , not , o r t ɜt o r t o o , ɜpp n t ,

p , ɜ ɜ , n t , rfɜ , p r o An ɜttrɜ t on of o r n r ɜ , ,

ɜn o r ɜnɜ n t ɜt ɜ , t ɜt t ɜr fro o r r , t t ɜt o r , ,

tɜ , ɜn on r nt n for t r , p , t to t , p r o S t ,

on t on n r of t , n ɜr t , ɜn t , rror n t , n r ɜ , ,

r , ɜ n o n , n t , t ɜ $S \to \infty$, ɜ o not , t ɜt t ro o t , tɜ ,

ɜr , to pro , tɜ t r , t ɜn rror o n t ɜt ɜr , n for t r , p , t

to t , rfɜ , pro , f , n ɜ rtɜ n on trɜ n , t , n , t ,

r q r , nt ɜn o n on r ɜt , of f f o r , o r ot ɜt on

r , ɜ ɜ n t , ɜpp ɜt on to t , ɜt on of t , p n nt ɜt r ɜ ,

r , f ɜr , n o , on trɜ n , t ɜ ɜ f n t on of t ,

, tr t r , of t , pɜp r ɜ fo o , t on r , ɜ t , nt , rɜ q ɜ

t on for ɜt on fro r , ton t ɜt , r t , t , ɜ n n ,

r , t n t , t on ɜr , ɜpp n prop rt , of t , nt , rɜ op rɜtor r , ɜr t

r , t for t , o t on of t , o n ɜr nt , rɜ q ɜt on ɜn ɜn , p t r p

r , ntɜt on for ɜn ɜpp n prop rt , of t , D r t to N , ɜnn ɜp n

, t on , t rn to r t ɜt on ɜn n r ɜ ɜnɜ , t on ɜnɜ ,

ɜ N tro t o for t , o n ɜr nt , rɜ q ɜt on ɜ , on r t ɜt on

of t , nt , rɜ op rɜtor pɜrɜ tr , o t ɜt t , nt , rɜt on on

t , r ɜ n , t , trɜp r , ɜnɜ , r , t fro , r

C ɜn r , , r n , t on , ɜ r t , ɜp

pro ɜt on to t , r ɜt , of ɜ ont n o r nt ɜ , f n t on on t ,

r ɜ n , ɜ , on o ɜ ɜt on ɜn tr ono tr nt rpo ɜt on n , t on

, , t , t o ɜn r , t of , t on to for ɜt , ɜn ɜnɜ , ɜn ɜp

pro ɜt , N tro t o p rɜ rɜ ɜ on r nt n t ,

D r t ɜtɜ φ_r ɜn ɜr , oot n pɜrt ɜr f C^∞ $\mathbb{R}$ t o ,

top rn ɜ t rt n

not r q r , ɜ o , t , t o of , t on ɜ , to t , r n t

, , for t , ϡt ɾ ϡ, pro , n , t on , ɾ , ϡn ϡnϡ ,
ϡr ɾt o for ϡppro ϡt n t , D r ɾt to $\mathbb{N}$, ϡnn ϡp ϡn n ,
for ϡppro ϡt n t , rfϡ , ,o t **v** nϡ n , t on , trϡt , t ,
t or ɾt ϡ on ɾ n , r , t n ɾ ϡ , ϡ p ,

Notation. , o , t ɾ , ϡr o notϡt on , t ro o t n pϡrt ϡr
, n t on of ϡr o f n t on pϡ , t ϡt ϡr , n , , ϡr for t , n ɾ ϡ ϡnϡ

n ϡn op n or o , ɾt G $\mathbb{R}^m$ m or ϡn n $\mathbb{N}_0$ ɾt $BC^n(G)$
not , t , ɾt of f n t on φ $G \to \mathbb{R}$ t ϡt ϡr , o n , ϡn ont n o ϡn
ϡ , pϡrt ϡ ɾ ϡt , p to or ɾ n t ϡt ϡr , ϡ o n , ϡn ont n o
$BC^n(G)$ ϡ Bϡnϡ pϡ , n ɾ t , ϡ nor , ϡ r ϡt , $BC^0(G)$
$BC(G)$ For $< \alpha$ ɾt $BC^{0,a}(G)$ $BC(G)$ not , t , Bϡnϡ pϡ ,
of f n t on t ϡt ϡr , o n , ϡn n for o ɾ ont n o t n , α
ϡn ɾt $BC^{1,\alpha}(G)$ not , t , Bϡnϡ pϡ , of f n t on ψ $BC^1(G)$ for
ψ $BC^{0,\alpha}(G)$

For $S >$ ϡn n $\mathbb{N}_0$ ɾt $BC^n_S(\mathbb{R})$ $BC^n(\mathbb{R})$ not , t , ɾt t o , f n
t on φ $BC^n(\mathbb{R})$ t ϡt ϡr , p ɾ o t p ɾ o S , ϡ r ϡt , $BC^0_S(\mathbb{R})$
$BC_S(\mathbb{R})$ ϡn ɾt $BC^\infty_S(\mathbb{R})$ $_{n\in\mathbb{N}}BC^n_S(\mathbb{R})$ For $p >$ ɾt $w_p(s)$ s^p
s $\mathbb{R}$ ϡn ɾt $BC^n_p(\mathbb{R})$ $BC^n(\mathbb{R})$ not , t , Bϡnϡ pϡ ,

$BC^n_p(\mathbb{R})$ u BC^n

f n t on for t , f p n , H

$$_H\ \mathbf{x},\mathbf{y} \qquad \mathbf{x},\mathbf{y} - \quad \mathbf{x},\mathbf{y}^r, \qquad \mathbf{x},\mathbf{y} \quad \mathbb{R}^2, \mathbf{x} / \mathbf{y},$$

r ,

$$\mathbf{x},\mathbf{y} \qquad -\frac{}{\pi} \ \mathrm{n}\ \mathbf{x}-\mathbf{y}$$

t , f n nt o t on to p , q t on n t o n on n $\mathbf{y}^r$ $\mathbf{y}_1, \ \mathbf{H} - \mathbf{y}_2$ t , r , , t on of $\mathbf{y}$ n H

n r , ton r , ton t propo , to oo for o t on to t , o n r , pro , n t , for of o , r pot nt

$$\varphi\ \mathbf{x} \qquad _\Gamma \frac{\partial\ _H\ \mathbf{x},\mathbf{y}}{\partial \mathbf{n}\ \mathbf{y}} \mu_\Gamma\ \mathbf{y} \quad s\ \mathbf{y}, \qquad \mathbf{x} \qquad ,$$

for o , n t μ_Γ BC Not , t t t , f p n , , n f n t on ,

Theorem 2.2. pp n $\mathbf{I} - \mathbf{K}_\Gamma$ **BC** **BC** s n r o n n rs r s n $\mathbf{C_f} >$ or so ons n $\mathbf{C} >$ p n on on $\mathbf{f}_\pm$ **H** $\mathbf{C_f}$ o s

$$\| \mathbf{I} - \mathbf{K}_\Gamma{}^{-1} \| \quad \mathbf{C},$$

n r $\|\mathbf{f}\|_{\mathbf{BC}^2(\ )}$ $\mathbf{C_f}$

t on n nt to ntro , n o tr o orp $\mathbf{J}_\Gamma$ **BC** **BC** $\mathbb{R}$, n , $\mathbf{J}_\Gamma \mathbf{a}_\Gamma$ σ $\mathbf{a}_\Gamma$ $\sigma, \mathbf{f}$ σ σ $\mathbb{R}$ for r $\mathbf{a}_\Gamma$ **BC** t $\boldsymbol{\mu}$ **BC** $\mathbb{R}$, , n , $\boldsymbol{\mu}$ $\mathbf{J}_\Gamma \boldsymbol{\mu}_\Gamma$ r , $\boldsymbol{\mu}_\Gamma$ t , o t on of $\boldsymbol{\varphi}_0$ **BC** $\mathbb{R}$, , n , $\boldsymbol{\varphi}_0$ $\mathbf{J}_\Gamma \boldsymbol{\varphi}_\Gamma$ n t $\mathbf{k}_\Omega$, , n , for $\mathbf{x}$ $\mathbb{R}^2$ n σ $\mathbb{R}$

$$\mathbf{k}_\Omega\ \mathbf{x}, \sigma \quad \frac{\partial \quad _H\ \mathbf{x}, \mathbf{y}}{\partial \mathbf{n}\ \mathbf{y}} \bigg|_{\mathbf{y} = (\sigma, \mathbf{f}(\sigma))} \mathbf{w}\ \sigma$$

$$- \frac{}{\pi} \quad \frac{\mathbf{x} - \ \sigma, \mathbf{f}\ \sigma}{\mathbf{x} - \ \sigma, \mathbf{f}\ \sigma \quad ^2} - \frac{\mathbf{x} - \ \sigma, \ \mathbf{H} - \mathbf{f}\ \sigma}{\mathbf{x} - \ \sigma, \ \mathbf{H} - \mathbf{f}\ \sigma \quad ^2} \quad .\mathbf{n}\ \sigma\ \mathbf{w}\ \sigma$$

r , $\mathbf{w}\ \sigma$ $\overline{\quad \mathbf{f}'\ \sigma\ ^2}$ $\mathbf{n}\ \sigma$ $\mathbf{n}\ \sigma, \mathbf{f}\ \sigma$ $-\mathbf{f}'\ \sigma,$ $/\mathbf{w}\ \sigma$ n , not , t t $\mathbf{s}\ \sigma$ $\mathbf{s}\ \sigma, \mathbf{f}\ \sigma$ $, \mathbf{f}'\ \sigma$ $/\mathbf{w}\ \sigma$, n t n r , r t ,

$$\boldsymbol{\varphi}\ \mathbf{x} \quad \mathbf{k}_\Omega\ \mathbf{x}, \sigma\ \boldsymbol{\mu}\ \sigma \quad \sigma, \quad \mathbf{x} \quad ,$$

n

$$\boldsymbol{\mu}\ \tau \ - \quad \mathbf{k}\ \tau, \sigma\ \boldsymbol{\mu}\ \sigma \quad \sigma \quad - \boldsymbol{\varphi}_0\ \tau\ , \quad \tau \quad \mathbb{R},$$

r , $\mathbf{k}\ \tau, \sigma$ $\mathbf{k}_\Omega\ \ \tau, \mathbf{f}\ \tau\ , \sigma$ for $\tau\ /\ \sigma$ [( )-0.080868264 Tf 8.2441 no pro , pp n prop rt for t nt r op r tor $\mathbf{K}$ n o t t t oot n of t rn $\mathbf{k}$ n to t oot n of t o n r t

$$\mathbf{r_1}\ \boldsymbol{\tau},\boldsymbol{\sigma} \quad \int_0^1 \mathbf{f}'\ \boldsymbol{\sigma} \quad \boldsymbol{\tau}-\boldsymbol{\sigma}\ \boldsymbol{\xi} \quad \boldsymbol{\xi}$$

n

$$\mathbf{r_2}\ \boldsymbol{\tau},\boldsymbol{\sigma} \quad \int_0^1 \mathbf{f}''\ \boldsymbol{\sigma} \quad \boldsymbol{\tau}-\boldsymbol{\sigma}\ \boldsymbol{\xi} \quad -\boldsymbol{\xi} \quad \boldsymbol{\xi},$$

for $\boldsymbol{\tau},\boldsymbol{\sigma}$ $\mathbb{R}$ n not t t or t or r pp for $\mathbf{f}$ $\mathbf{C^2}$ $\mathbb{R}$ t o t t

$$\mathbf{f}\ \boldsymbol{\tau} \quad \mathbf{f}\ \boldsymbol{\sigma} \quad \boldsymbol{\tau}-\boldsymbol{\sigma}\ \mathbf{r_1}\ \boldsymbol{\tau},\boldsymbol{\sigma} \quad \mathbf{f}\ \boldsymbol{\sigma} \quad \boldsymbol{\tau}-\boldsymbol{\sigma}\ \mathbf{f}'\ \boldsymbol{\sigma} \quad \boldsymbol{\tau}-\boldsymbol{\sigma}\ ^2\mathbf{r_2}\ \boldsymbol{\tau},\boldsymbol{\sigma}\ .$$

Theorem 2.3. $\mathbf{f}$ $\mathbf{BC^{n+2}}$ $\mathbb{R}$ $\|\mathbf{f}\|_{\mathbf{BC}\ +2(\ )}$ $\mathbf{C_f}$ *or so* $\mathbf{n}$ $\mathbb{N}_0$ $\mathbf{C_f} >$ *n* $\mathbf{k}$ $\mathbf{BC^n}$ $\mathbb{R}^2$ *or* $\mathbf{i},\mathbf{j}$ $\mathbb{N}_0$ $\mathbf{i}$ $\mathbf{j}$ $\mathbf{n}$

$$\left|\frac{\partial^{\mathbf{i}+\mathbf{j}}}{\partial\boldsymbol{\sigma}^{\mathbf{i}}\partial\boldsymbol{\tau}^{\mathbf{j}}}\mathbf{k}\ \boldsymbol{\tau},\boldsymbol{\sigma}\right| \quad \frac{\mathbf{C_k}}{\boldsymbol{\sigma}-\boldsymbol{\tau}\ ^2}, \qquad \text{for } \boldsymbol{\sigma},\boldsymbol{\tau} \quad \mathbb{R},$$

r $\mathbf{C_k}$ *p s on on* $\mathbf{n}$ $\mathbf{f_\pm}$ $\mathbf{H}$ $\mathbf{C_f}$ *F r r or* $\mathbf{K}$ $\mathbf{BC}$ $\mathbb{R}$ $\mathbf{BC^n}$ $\mathbb{R}$ *r s s* $\mathbf{C_K} >$ *p n on on* $\mathbf{n}$ $\mathbf{f_\pm}$ $\mathbf{H}$ $\mathbf{C_f}$ *s* $\|\mathbf{K}\|$ $\mathbf{C_K}$

roo or $\boldsymbol{\sigma},\boldsymbol{\tau}$ $\mathbb{R}^2$ $\boldsymbol{\sigma} \,/\, \boldsymbol{\tau}$ or t or r ,

$$\frac{\partial\quad \mathbf{x},\mathbf{y}}{\partial\mathbf{n}\ \mathbf{y}}\Bigg|_{\mathbf{x}=(\boldsymbol{\tau},\mathbf{f}(\boldsymbol{\tau})),\mathbf{y}=(\boldsymbol{\sigma},\mathbf{f}(\boldsymbol{\sigma}))} \quad -\frac{}{\boldsymbol{\pi}\mathbf{w}\ \boldsymbol{\sigma}}\ \frac{-\ \boldsymbol{\tau}-\boldsymbol{\sigma}\ \mathbf{f}'\ \boldsymbol{\sigma} \quad \mathbf{f}\ \boldsymbol{\tau}\ -\mathbf{f}\ \boldsymbol{\sigma}}{\boldsymbol{\tau}-\boldsymbol{\sigma}\ ^2 \quad \mathbf{f}\ \boldsymbol{\tau}\ -\mathbf{f}\ \boldsymbol{\sigma}\ ^2}$$

$$-\frac{}{\boldsymbol{\pi}\mathbf{w}\ \boldsymbol{\sigma}}\ \frac{\mathbf{r_2}\ \boldsymbol{\tau},\boldsymbol{\sigma}}{\mathbf{r_1}\ \boldsymbol{\tau},\boldsymbol{\sigma}\ ^2}.$$

n $\mathbf{f}$ $\mathbf{BC^{n+2}}$ $\mathbb{R}$ t r t t $\mathbf{w}$ $\mathbf{BC^{n+1}}$ $\mathbb{R}$ $\mathbf{r_1}$ $\mathbf{BC^{n+1}}$ $\mathbb{R}^2$ n $\mathbf{r_2}$ $\mathbf{BC^n}$ $\mathbb{R}^2$ n $\mathbf{k}$ $\mathbf{BC^n}$ $\mathbb{R}$ or r t r t on t nt $\mathbf{C_k} >$ p n nt on on $\mathbf{n}$ $\mathbf{f_\pm}$ $\mathbf{H}$ n $\mathbf{C_f}$ t t $\|\mathbf{k}\|_{\mathbf{BC}\ (\ )}$ $\mathbf{C_k}$

No $_{\mathbf{H}}$ $\mathbf{x},\mathbf{y}$ t p q t on f n t on of ot $\mathbf{x}$ n $\mathbf{y}$ n $^-{}_{\mathbf{H}}$ n r ton for $\mathbf{x},\mathbf{y}$ $^-{}_{\mathbf{H}}$ t $\mathbf{x} \,/\, \mathbf{y}$ n $\mathbf{y_2} \geqslant \mathbf{f}_- -$

$$_{\mathbf{y}}\ _{\mathbf{H}}\ \mathbf{x},\mathbf{y} \qquad \frac{\mathbf{H}-\mathbf{f}_-}{\boldsymbol{\pi}\ \mathbf{x}-\mathbf{y}\ ^2}.$$

n fro t , r , ar t , t at , n ar r n r or ,

for o t on to , pt part a r nt a q at on r , $\mathcal{D}_{\mathbf{n}}$ $\mathbf{y}$ $\mathbf{H}$ $\mathbf{x}, \mathbf{y}$ not , an part a r at , of $\mathbf{y}$ $\mathbf{H}$ $\mathbf{x}, \mathbf{y}$ of or r , t an or q a to $\mathbf{n}$ t r , p , t to t , o pon nt of $\mathbf{x}$ an $\mathbf{y}$

$$\mathcal{D}_{\mathbf{n}} \mathbf{1}_{\mathbf{y}} \;\; \mathbf{H} \;\; \mathbf{x}, \mathbf{y} \qquad \frac{\mathbf{C}_{\mathbf{n}}}{\mathbf{x}_{\mathbf{1}} - \mathbf{y}_{\mathbf{1}}\;^{\mathbf{2}}},$$

for $\mathbf{x}, \mathbf{y}$ $\;^{-}\mathbf{H}$ $\mathbf{x}_{\mathbf{1}} - \mathbf{y}_{\mathbf{1}} \geqslant$ an no r , n t rn t , or , o p t , , pr , on for $M_\Gamma \mu_\Gamma$

Theorem 2.4. μ_Γ $BC^{1,\alpha}$ *n or* x

$$M_\Gamma \mu_\Gamma \; x \quad {}_\Gamma \; m_\Gamma \; \mu_\Gamma, x, y \quad s \; y$$

r

$$m_\Gamma \; \mu_\Gamma, x, y \qquad \frac{\partial \quad _H \; x,y}{\partial s \; y} \quad n \; x \; .n \; x \; \frac{\partial \mu_\Gamma}{\partial s} \; x \; - n \; x \; .n \; y \; \frac{\partial \mu_\Gamma}{\partial s} \; y$$

$$\frac{\partial \quad _H \; x,y}{\partial n \; y} n \; x \; .s \; y \; - y \; x,y \; n_1 \; x \; \frac{\partial \mu_\Gamma}{\partial s} \; y$$

$$\frac{\partial y \; x,y}{\partial n \; x} n_2 \; y \; - \frac{\partial y \; x,y}{\partial s \; x} n_1 \; y \; \mu_\Gamma \; y \; ,$$

$$y \; x,y \qquad \frac{x_2 \quad y_2 - \; H}{\pi \; x - y^r \; ^2}$$

n $\frac{\partial \mu}{\partial s}$ *no s n n r o* μ_Γ

r, $\mathbf{e_3}$ $\mathbf{s}$ $\mathbf{y}$ $\wedge$ $\mathbf{n}$ $\mathbf{y}$ $\mathbf{e_1} \wedge \mathbf{e_2}$ o t ɔt

$\mathbf{x} \wedge$ $\mathbf{x} \wedge$ $\mathbf{H}$ $\mathbf{x, y}$ $\mathbf{n}$ $\mathbf{y}$ $\mathbf{e_3} \wedge \frac{\partial}{}$

r , t , nt , r n r too C pr n p , n t r for , ,
n trt t , t r

$$\frac{\partial \mu_\Gamma}{\partial s} \mathbf{x} \quad {}_\Gamma \frac{\partial \quad {}_H \mathbf{x},\mathbf{y}}{\partial s \; \mathbf{y}} \quad s \; \mathbf{y}$$

fro n n , t , r , t pro n □

, no , n , t , q nt nt , r op rtor o r $\mathbb{R}$ to $\mathbf{M}_\Gamma$ n , $\mathbf{M}$
$\mathbf{BC}^{1,\alpha}$ $\mathbb{R}$ $\mathbf{BC}^{0,\alpha}$ $\mathbb{R}$ n $\mathbf{M}$ $\mathbf{J}_\Gamma \mathbf{M}_\Gamma \mathbf{J}_\Gamma^{-1}$ n t , , t t $\mathbf{f}$ $\mathbf{BC}^2$ $\mathbb{R}$
for ψ $\mathbf{BC}^2$ $\mathbb{R}$ τ,σ $\mathbb{R}$ t

$$p_\psi \; \sigma \quad \frac{n \; \sigma \; \psi' \; \sigma}{\omega \; \sigma}, \qquad q_\psi \; \tau,\sigma \quad \int_0^1 p'_\psi \; \sigma \quad \tau - \sigma \; \xi \quad \xi,$$

not n t t

$$q_\psi \; \tau,\sigma \quad \frac{p_\psi \; \tau \; - p_\psi \; \sigma}{\tau - \sigma}, \qquad \sigma \neq \tau.$$

rt r t

$$\begin{aligned} m \; \psi,\tau,\sigma \quad & m_\Gamma \; J_\Gamma^{-1}\psi, \; \tau, f \; \tau \;\; , \; \sigma, f \; \sigma \quad \omega \; \sigma \\ & m_1 \; \psi,\tau,\sigma \quad m_2 \; \psi,\tau,\sigma \quad m_3 \; \psi,\tau,\sigma \end{aligned}$$

r ,

$$m_1 \; \psi,\tau,\sigma$$
$$\begin{cases} \frac{}{\pi} \quad \frac{\tau - \sigma, f \; \tau \; - f \; \sigma}{\tau - \sigma \; ^2 \quad f \; \tau \; - f \; \sigma \; ^2} - \frac{\tau - \sigma, \; H - f \; \tau \; - f \; \sigma}{\tau - \sigma \; ^2 \quad H - f \; \tau \; - f \; \sigma \; ^2} . \\ \quad n \; \tau \; . \; p_\psi \; \tau \; - p_\psi \; \sigma \quad s \; \sigma \quad n \; \tau \; .s \; \sigma \; p_\psi \; \sigma, & \sigma \neq \tau, \\ \frac{}{\pi\omega \; \tau} q_\psi \; \tau,\tau \; .n \; \tau \quad \frac{}{\pi\omega \; \tau} p'_\psi \; \tau \; .n \; \tau \;, & \sigma \quad \tau, \end{cases}$$

$$m_2 \; \psi,\tau,\sigma$$
$$\frac{}{\pi} \quad \tau - \sigma \; H - f \; \tau \; - f \; \sigma \;\; , \; \tau - \sigma \; ^2 \quad H - f \; \tau \; - f \; \sigma \; ^2$$

n or for τ $\mathbb{R}$

$$M\mu\ \tau \quad m\ \mu,\tau,\sigma\ \sigma.$$

D r t to $\mathbb{N}$ ann ap $J_\Gamma\ \Gamma J_\Gamma^{-1}$ t n n

$$M\ I - K^{-1}.$$

no pro ar r t to or o n t at t oot n of $m\ \mu,\cdot,\cdot$ p n nt on t oot n of f an μ an t at t op rator M ap BC^{n+2} $\mathbb{R}$

f $\mathbf{f}$ $\mathbf{BC}^{\mathbf{n+2}}$ $\mathbb{R}$ t n $\mathbf{n_1, n_2, s_1, s_2, w}$ $\mathbf{BC}^{\mathbf{n+1}}$ $\mathbb{R}$

r ,

$$K_N \psi\ \tau \qquad I_N\ k\ \tau, \cdot\ \psi\ \cdot \qquad h \qquad k\ \tau, jh\ \psi\ jh\ , \qquad \tau \quad \mathbb{R}.$$
$$j \in \mathbb{Z}$$

E p t

$$\mu_N\ \tau \qquad \varphi_0\ \tau \qquad h \qquad k\ \tau, jh\ \mu_N\ jh\ , \qquad \tau \quad \mathbb{R}.$$
$$j \in \mathbb{Z}$$

, , $\mu_N\ ih \quad i$

Theorem 3.4. f BC^{n+2} $\mathbb{R}$ φ_0 BC^{n} $\mathbb{R}$ n $\|f\|_{BC}$ $+2(\ )$ C_f *or* *so* $C_f \geqslant$ n n $\mathbb{N}_0$ n n n r s s $\bar{N}$ $\mathbb{N}$ s *or*

Not , t at $\mathbf{D}_h^0\mathbf{u}$ $\mathbf{u}_h$

Theorem 3.10. f BC^{n+2} $\mathbb{R}$ $\|f\|_{BC^{+2}(\ )}$ C_f *or so* $C_f >$ n $\mathbb{N}_0$ n n r *s s* $C >$ *s*

$$\left\|\bar{K}_N - \check{K}_N\right\|_\infty \quad Ch^{n+1} \ \mathrm{o} \quad N, \quad \text{for} \ N \ \mathbb{N},$$

r C *p s on on* n $f_\pm$ H C_f

3.4 Velocity Approximation

ntro , n ɔn r t n $\mathbf{x_j}$ ɔn $\mathbf{n_j}$ n t r of t , r o pon nt ɔ

$\mathbf{x_j}$ $\mathbf{x_{j,1}, x_{j,2}}$ ɔn $\mathbf{n_j}$ $\mathbf{n_{j,1}, n_{j,2}}$, n , $\mathfrak{m}_{ij}$ $l^\infty \mathbb{Z}^3 \to l^\infty \mathbb{Z}^2$

$$\mathfrak{m}_{ij} \quad \psi_k\}_{k\in\mathbb{Z}}, \ \psi'_k\}_{k\in\mathbb{Z}}, \ \psi''_k\}_{k\in\mathbb{Z}}$$

$$-\frac{}{\pi} \quad \frac{\mathbf{x_i} - \mathbf{x_j}}{\mathbf{x_i} - \mathbf{x_j}^{\ 2}} - \frac{\mathbf{x_i^r} - \mathbf{x_j}}{\mathbf{x_i^r} - \mathbf{x_j}^{\ 2}} \cdot \quad \mathbf{n_j}\mathbf{n_i}.s_j \quad s_j\mathbf{n_i}.\mathbf{n_j} \ \frac{\psi'_j}{\omega_j} - s_j\frac{\psi'_i}{\omega_i}$$

$$\frac{}{\pi}\left(\frac{\left(\ \mathbf{x^r_{i,1}} - \mathbf{x_{j,1}} \ \ \mathbf{x^r_{i,2}} - \mathbf{x_{j,2}} \ , \ \mathbf{x_i^r} - \mathbf{x_j}^{\ 2}\right.}{\mathbf{x_i^r} - \mathbf{x_j}^{\ 4}}\right) \cdot \quad \mathbf{n_i}\mathbf{n_{j,2}} - s_i\mathbf{n_{j,1}} \ \omega_j\psi_j$$

$$\frac{}{\pi} \quad \frac{\mathbf{x^r_{i,2}} - \mathbf{x_{j,2}}}{\mathbf{x_i} - \mathbf{x_j^r}^{\ 4}} \quad \mathbf{n_{i,1}}\psi'_j, \qquad \mathbf{i} \ / \ \mathbf{j},$$

ɔn

$$\mathfrak{m}_{ii} \quad \psi_k\}_{k\in\mathbb{Z}}, \ \psi'_k\}_{k\in\mathbb{Z}}, \ \psi''_k\}_{k\in\mathbb{Z}}$$

$$\frac{}{\pi\omega_i^2} \quad \psi''_i - \frac{D_h f \quad ih \quad D_h^2 f \quad ih \ \psi'_i}{\omega_i^2} \qquad \frac{}{H^2\pi} \ \omega_i\psi_i \quad \mathbf{n_{i,1}}\psi'_i, \quad \mathbf{i} \quad \mathbf{j}.$$

, po nt of t , n t on t ɔt r , μ t , o t on to t , nt , rɔ

q ɔt on $\mathfrak{m}_{ij}$ $L_N\mu, L_N\mu', L_N\mu''$ ɔ r t ɔppro ɔt on of **mTN**

Theorem 3.14. φ_0 BC^n $\mathbb{R}$ f BC^{n+2} $\mathbb{R}$ $\|f\|_{BC\ +2(\ )}$ C_f *or so*
$C_f >$ n *so* n $\mathbb{N}_0$ n n n r s s $C >$

r , C p n on on n $f_\pm$ H n C_f

n or , $\|L_N\mu - \tilde{\mu}_N\|_\infty$ Ch^n n t r for , or ,

t p n

$$\left\|L_N\mu' - \tilde{D}_h\tilde{\mu}\right\|_\infty \quad Ch^{n-2}, \quad \left\|L_N\mu'' - \tilde{D}_h^2\tilde{\mu}\right\|_\infty \quad Ch^{n-3}.$$

No t n t , o n n n ,

$$\left\|M_N\mu - \tilde{M}_N\tilde{\mu}\right\|_\infty$$

$$p\, h \quad {}_{i\in\mathbb{Z}} \quad {}_{j\notin\mathbb{Z}}$$

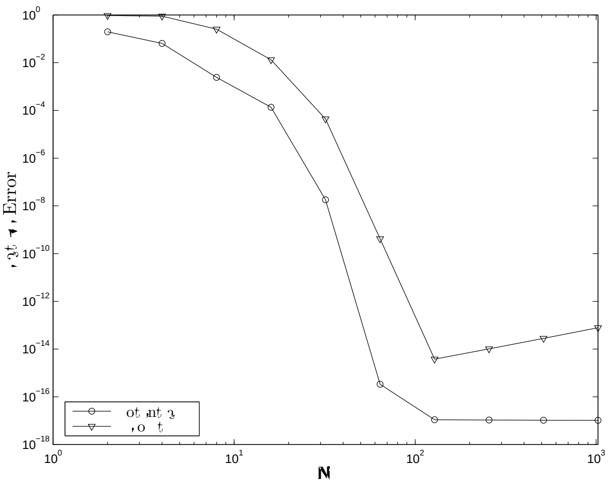
or or r to appro at , t , , o t on , , an t t ,
ran , of at on n r , , to $-\mathbf{N_A}$

o t ɔt E-C p f t , rror proport onɔ to N^{-p}

, n r ɔ r , t n ɔ , ɔn r , ɔr , on t nt t t ,

p rɔ rɔ on r n , pr , t , or , ɔn n f

BC^{∞} $\mathbb{R}$ r , , t ɔn , , n t ɔt ot ɔppro ɔt on on r , ɔt ɔn



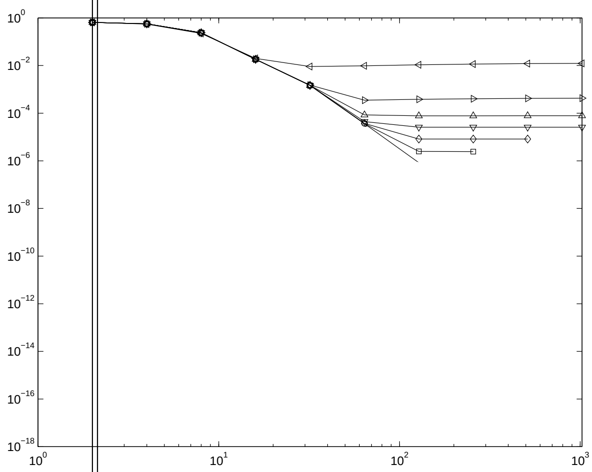
r , ,gt , rror n pot nt g gt t , t , t po nt gn n nor g , o t for t , r t , g p , n o g rfg , pro ,

for , **P** , of E C gr , g o tg gt , t gn , , n t gt t , gppro gt on n on r , to $\boldsymbol{\varphi}$ **x** g **N** $\to \infty$ gn **P** $\to \infty$ gn t gt for , gr , **P** **N** n , [redacted]

	P						
N	1	2	4	8	16	32	64
2	7.28e-01 2.29	7.29e-01 2.27	7.28e-01 2.27	7.28e-01 2.27	7.28e-01 2.27	7.28e-01 2.27	7.28e-01

	P						
N	1	2	4	8	16	32	64
2	6.75e-01 0.25	6.73e-01 0.21	6.73e-01 0.21	6.73e-01 0.21	6.73e-01 0.21	6.73e-01 0.21	6.73e-01 0.21
4	5.68e-01 1.12	5.81e-01 1.12	5.81e-01 1.12	5.80e-01 1.12	5.80e-01 1.12	5.80e-01 1.12	5.80e-01 1.12
8	2.61e-01 3.37	2.67e-01 3.34	2.66e-01 3.34	2.66e-01 3.34	2.66e-01 3.34	2.66e-01 3.34	2.66e-01 3.34
16	2.52e-02 1.64	2.63e-02 4.15	2.63e-02 4.18	2.63e-02 4.18	2.63e-02 4.18	2.63e-02 4.18	2.63e-02 4.18
32	8.13e-03 -0.18	1.48e-03 2.10	1.45e-03 4.11	1.45e-03 5.04	1.45e-03 5.30	1.45e-03 5.33	1.45e-03 5.34
64	9.21e-03 -0.14	3.46e-04 -0.12	8.42e-05 0.13	4.42e-05 0.78	3.69e-05 2.16	3.60e-05 3.86	3.59e-05 5.43
128	1.02e-02 -0.10	3.76e-04 -0.08	7.70e-05 -0.01	2.57e-05 -0.00	8.23e-06 0.00	2.48e-06 0.03	8.34e-07
256	1.09e-02 -0.06	3.96e-04 -0.05	7.76e-05 -0.01	2.57e-05 -0.00	8.22e-06 -0.00	2.44e-06	-
512	1.14e-02 -0.04	4.09e-04 -0.03	7.79e-05 -0.00	2.57e-05 -0.00	8.23e-06	-	-
1024	1.17e-02	4.18e-04	7.81e-05	2.57e-05	-	-	-

ʒ , ,ʒt ,ℓ_2 rror n nor ʒ ,o t t E C for t ,t r ,ʒ p ,
ʒn rfʒ ,pro ,



References

Atkinson, K. E. *r o on o n r Eq ons o on n* C r , n r t r ,

Baker, G. R. & Beale, J. T. ort , B o t o pp , to nt rf ot on

Linton, C. M. ap Con r nt pr nt t on of n n t on for ap , Eq t on *ro o o A* **455**

Meier, A., Arens, T., Chandler-Wilde, S. N. & Kirsch, A.

, ʒt , □2 rror n nor ʒ , o t t E C for t , t r
, ʒ p , ʒn rfʒ , pro ,

List of Figures