Boundary integral methods in high frequency scattering

[illegible]

Abstract

In this article we review recent progress on the design, analysis and implementation of numerical-asymptotic boundary integral methods for the computation of frequency-domain acoustic scattering in a homogeneous unbounded medium by a bounded obstacle. The main aim of the methods is to allow computation of scattering at arbitrarily high frequency with finite computational resources.

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d c n o o n c on oy y o c n y o o n n y c n o on
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o o o n y do y d o n do
y nd y n o o **S D D** nd **H** d n d c y y

$$\mathbf{S}\psi \quad \mathbf{x,y}\ \psi\ \mathbf{y}\ ds\ \mathbf{y}\ , \qquad \mathbf{D}\psi \quad \frac{\partial \quad \mathbf{x,y}}{\partial n\ \mathbf{y}}\ \psi\ \mathbf{y}\ ds\ \mathbf{y}$$

$$\mathbf{D}'\psi \quad \frac{\partial \quad \mathbf{x,y}}{\partial n\ \mathbf{x}}\ \psi\ \mathbf{y}\ ds\ \mathbf{y}\ , \qquad \mathbf{H}\psi \quad \frac{\partial^2 \quad \mathbf{x,y}}{\partial n\ \mathbf{x}\ \partial n\ \mathbf{y}}\ \psi\ \mathbf{y}\ ds\ \mathbf{y}\ .$$

c on c n n n

$$\mathbf{A}'\mathbf{v} \quad \mathbf{I} \quad \mathbf{D}' - \eta \mathbf{S}\ \mathbf{v} \quad \mathbf{f}\ , \qquad \mathbf{v} \quad \partial u/\partial n\ .$$

n on o on no n nd d oy c]] d] d

on cond nd n on c d n n no n o on
$\mathbf{v}$ $\frac{}{n}$ nd on on o n o nd y
c n y o $\mathbf{C}^1$ c n n o o $\mathbf{D}$ nd $\mathbf{S}$ n co c
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c n n nd co oc on od n c o yno nc on
d o n y d n d n c o on $\mathbf{v}_N$ y n o o
o o

$$\|\mathbf{v} - \mathbf{v}_N\| \leq \mathbf{C} \inf_{\mathbf{N} \in \mathbf{S}_N} \|\mathbf{v} - \boldsymbol{\varphi}_N\|,$$

$\mathbf{S}_N$ d no n d n on o on c n d nd $\mathbf{N}$ d
c on d n on c $\mathbf{S}_N$ Mo c y o o y
d n d n od nd co oc on od c c n A n on
$\mathbf{C} >$ nd $\mathbf{N}_0 >$ c od o $\mathbf{N} \geq \mathbf{N}_0$
§ o d
B d on on c n n n c n y o od o c n
o n c on d on

d n ood $\mathbf{k}$ d nd n n d n on o on c $\mathbf{S}_N$ o
o on o $\inf_{\mathbf{N} \in \mathbf{S}_N} \|\mathbf{v} - \boldsymbol{\varphi}_N\|$ o n o y o
$\mathbf{k} \to \infty$ c no y d nd on $\mathbf{k}$ nd o d no $\mathbf{S}_N$

oo o o d nd nc y con n $\mathbf{C}$ n on
$\mathbf{k}$ o y o n n nd c o nd dn o d o $\mathbf{k} \to \infty$

d n ood od n n n c od n o
o on c n d y o n co on
c n o nd d $\mathbf{k} \to \infty$

o n d n $\mathbf{v}$ o on o
on o o d n con n o c d $\mathbf{N}$ $\mathbf{k} \to \infty$ c n on n y
o n n §
o c c o n y o cond nd n on
od o c n y $\mathbf{N}$ $\mathbf{N} \geq \mathbf{N}_0$ o c n $\mathbf{k}$
non n y n d n o o $\mathbf{A}$ n y no c n
n o on on o o d $\mathbf{N}$ con n $\mathbf{C}$ d nd on $\mathbf{k}$ o
o o d $\mathbf{C}$N

o o o con n **B** nd **α** n on n y o Mo
n on od o o on d o n **any** n d n on
c $\mathbf{S}_N$ $\mathbf{L}_2$ $\mathbf{v}_N$ $\mathbf{S}_N$ c

$$\mathbf{a}\ \mathbf{v}_N, \mathbf{w}_N \quad \mathbf{f}, \mathbf{w}_N\ _{L_2(\Gamma)}, \quad \text{o} \quad \mathbf{w}_N\ \mathbf{S}_N,$$

n o **C** **B/α** o on o n y o n

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n **x** **· a** [illegible] A **k → ∞** n [illegible] on on co [illegible] oc [illegible] d [illegible] o nd o n

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n c on $\mathbf{C}^{\infty}$ con con o nd n n on
n y d d d y o n ncy o n $\mathbf{T}_1$ nd $\mathbf{T}_2$ n o n n d zon
nd do zon d c d n L n γ , $\boldsymbol{\pi}$ $\rightarrow$ $\boldsymbol{\pi}$ od c
z on o d n $\mathbf{t}$, $\boldsymbol{\pi}$ o o $\mathbf{T}$ γ $\mathbf{t}$ $\mathbf{T}$ o
$\mathbf{i}$, o c c n z y n n n $\mathbf{v}$ γ $\mathbf{s}$, $\mathbf{k}$
$\mathbf{v}$ $\mathbf{s}, \mathbf{k}$ o con n nc

$$\mathbf{v}\ \mathbf{s},\mathbf{k} \quad \mathbf{k} \quad \mathbf{k}\gamma\ \mathbf{s} \cdot \mathbf{a}\ \mathbf{V}\ \mathbf{s},\mathbf{k}\ .$$

n c , $\boldsymbol{\pi}$ co d y o n , $\mathbf{i}$, . . . , $_1$
nd $_2$ n y n o ood o n ncy o n $\mathbf{t}_1$ nd $\mathbf{t}_2$ nd $_3$ nd $_4$

on d n n c n y o od c o y on nd d n § o n n d o d o V s, k c o s c c n k n n d nd do zon Mo n d o on n d c y o V n d do zon 1 n dy o o y o o c o c o on n o o n n d

For all L, M ∈ ℕ ∪ { }, the function V(s, k) admits a decomposition of the form:

$$\mathbf{V}(s,k) \quad \left[\sum_{=0}^{L\,M} k^{-1\;3-2\;\;3-} \; b \quad (s) \quad (\,) \; (k^{1\;3} Z(s)) \right] \quad \mathbf{R}_{L\,M}(s,k) \;,$$

for s ∈ , π , where the remainder term has its nth derivative bounded, for n ∈ ℕ ∪ { }, by

$$|\mathbf{D}^n \mathbf{R}_{L\,M}(s,k)| \leq \mathbf{C}_{L\,M\,n} \quad k^{\;\;+n\;3} \;,$$

where μ − n $\frac{2}{3}$ L , M and $\mathbf{C}_{L,M\,n}$ is independent of k. The functions b and Z are C^∞ π-periodic functions. Z has simple zeros at t_1 and t_2

For all $n \in \mathbb{N} \cup \{\ \}$ there exist constants $C_n >$ independent of k and $s \in\ , \pi$, such that for all k sufficiently large,

$$|D^n V(s,k)| \le C_n \begin{cases} , & n \quad , , \\ k^{-1}(k^{-1\ 3} \quad |\omega(s)|)^{-n-2}, & n \ge\ , \end{cases}$$

where $\omega(s) \quad (s - t_1)(t_2 - s)$. These estimates are uniform in $s \in\ , \pi$.

y c oo n **p** o o y n $k^{1\ 9}$ cc cy o od
k nc N c n o
B o n d c on n on n y o c n **s** n
o n t_1 n do on n n on o n n
n y o c $k \to \infty$

$$k^{-1\ 3} b_{0\ 0} \quad k|Z\ s|^3 / \quad \alpha_1\ k^{1\ 3}|Z\ s| \quad - \alpha_1\ k^{1\ 3}|Z\ s| .$$

nc α_1 n d n o co n con n o o c
o y c o on o c n c **k** nd on c $k^{1\ 3}$ d d y on n y
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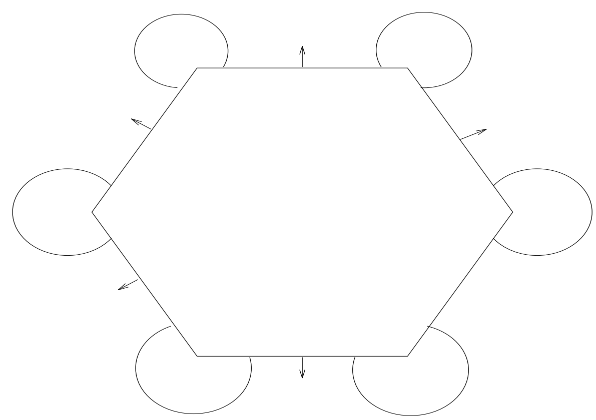

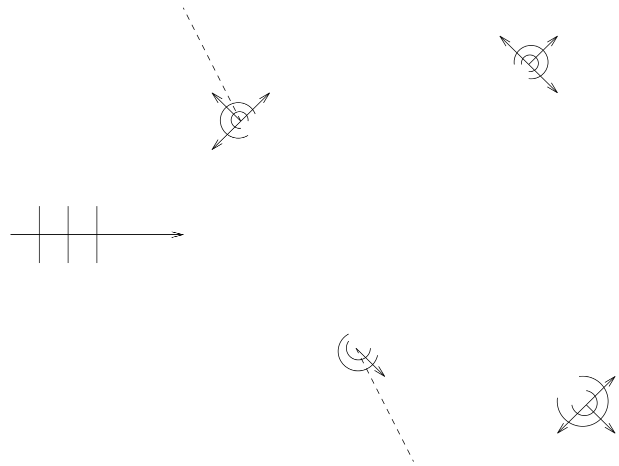

$$\mathbf{C} \leq \frac{\mathbf{B}}{\alpha}$$

$$|a(\mathbf{v},\mathbf{w})| = |(\mathbf{A}\mathbf{v},\mathbf{w})_{L^2(\Gamma)}| \leq \|\mathbf{A}\mathbf{v}\|_{L^2(\Gamma)} \|\mathbf{w}\|_{L^2(\Gamma)}$$
$$\leq \|\mathbf{A}\| \|\mathbf{v}\|_{L^2(\Gamma)} \|\mathbf{w}\|_{L^2(\Gamma)}.$$

$$\|\mathbf{A}\mathbf{v}\|_{L^2(\Gamma)} \|\mathbf{v}\|_{L^2(\Gamma)} \geq |(\mathbf{A}\mathbf{v},\mathbf{v})_{L^2(\Gamma)}| = |a(\mathbf{v},\mathbf{v})| \geq \alpha\|\mathbf{v}\|^2_{L^2(\Gamma)},$$

$$\|\mathbf{A}^{-1}\| \leq \alpha^{-1}.$$

condition number

$$\frac{\mathbf{B}}{\alpha} \geq \operatorname{cond} \mathbf{A} = \|\mathbf{A}\| \|\mathbf{A}^{-1}\|.$$

N → ∞ k

as $k \to \infty$, uniformly in $\eta >$.

n n o on n y nd d
n o n a n o

$$|S| \leq \sqrt{\frac{a}{\pi k}}.$$

An o n c on o o n n n o
A on η c o n o

$$l_1 R_0^{-1}, l_2 k \le \eta \le u_1 R_0^{-1}, u_2 k ,$$

o o o con n l_1 l_2 u_1 nd u_2 n o o con n $c >$ $\|A^{-1}\|$ $\|A^{-1}\| \le$ c o $k >$ o c oo n

$$\eta \quad R_0^{-1} \quad k,$$

c l_1 l_2

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d n on o **hybrid** o on n o n nd c n n o

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