On con ergence of dyn cs of hopp ng p rt cl es to rth nd de th process n cont n

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M rch , 8

Abstract

We show that some classes of birth-and-death processes in continuum

Here and below, for simplicity of notations, we just write $\mathbf{x}, \mathbf{y}$ instead of $\{\mathbf{x}\}$

Let us briefly recall some basic facts of harmonic analysis on the configuration space, see [,] for further details. Let Γ_0 denote the space of all finite configurations in $\mathbb{R}^d$, i.e., $\Gamma_0 := \bigsqcup_{n=0}^{\infty} \Gamma^{(n)}$, where $\Gamma^{(n)}$ is the space of all n-point configurations in $\mathbb{R}^d$. Clearly, $\Gamma_0 \subset \Gamma$, and we define $\mathcal{B}(\Gamma_0)$ and $\mathcal{B}(\Gamma^{(n)})$ as the trace σ-algebra of $\mathcal{B}(\Gamma)$ on Γ_0 and $\Gamma^{(n)}$, respectively. For a function $G : \Gamma_0 \to \mathbb{R}$, we define a function $(KG)(\gamma) := \sum_{\eta \Subset \gamma} G(\eta)$, $\gamma \in \Gamma$, provided the summation makes sense. Here $\eta \Subset \gamma$ means that η is a finite subset of γ.

Let μ be a probability measure on $(\Gamma, \mathcal{B}(\Gamma))$. Then there exists a unique measure ρ on $(\Gamma_0, \mathcal{B}(\Gamma_0))$ satisfying

$$\int_{\Gamma} (KG)(\gamma)\, \mu(d\gamma) = \int_{\Gamma_0} G(\eta)\, \rho(d\eta)$$

for each measurable function $G : \Gamma_0 \to [0, \infty]$. The measure ρ is called the correlation measure of μ. Further, denote by λ the Lebesgue–Poisson measure on Γ_0, i.e.,

$$\lambda := \delta_{\varnothing} + \sum_{n=1}^{\infty} \frac{1}{n!}\, dx_1 \cdots dx_n.$$

Here $\delta_{\varnothing}$ is the Dirac measure with mass at $\varnothing$, and $dx_1 \cdots dx_n$ is the Lebesgue measure on $\Gamma^{(n)}$, which is naturally defined on this space. Assume that the correlation measure ρ of μ is absolutely continuous with respect to λ. Then $k := \frac{d\rho}{d\lambda}$ is called the correlation functional of μ. For a given correlation functional k, the corresponding Ursell functional $u : \Gamma_0 \to \mathbb{R}$ is defined through the formula $k(\eta) = \sum_{\pi \in \mathcal{P}(\eta)} u_\pi(\eta)$, where $\mathcal{P}(\eta)$ denotes the set of all partitions of η, and given a partition $\pi = \{\eta_1, \dots, \eta_k\}$ of η, $u_\pi(\eta) := u(\eta_1) \cdots u(\eta_k)$. Recall also that a function $G : \Gamma_0 \to \mathbb{R}$ is called translation invariant if, for each $x \in \mathbb{R}^d$, $G(\eta_x) = G(\eta)$ for all $\eta \in \Gamma_0$, where η_x denotes the configuration η shifted by vector x, i.e., $\eta_x := \{y + x \mid y \in \eta\}$. Clearly, the correlation functional k is translation invariant if and only if the corresponding Ursell functional u is translation invariant.

If k is the correlation functional of a probability measure μ on Γ, we denote

$$k^{(n)}(x_1, \dots, x_n) := k(\{x_1, \dots, x_n\}), \quad n \in \mathbb{N},$$

and analogously we define $u^{(n)}$. The $(k^{(n)})_{n=1}^{\infty}$ and $(u^{(n)})_{n=1}^{\infty}$ are called the correlation and Ursell functions of μ, respectively. Note that, if k is translation invariant, then $k^{(1)} = u^{(1)}$ is a constant.

For a function $f : \mathbb{R}^d \to \mathbb{R}$, we define $e_\lambda(f, \eta) := \prod_{x \in \eta} f(x)$, $\eta \in \Gamma_0$

Ass e th$\mathbf{S}$t $\mathbf{L}$ is $\mathbf{S}$ M$\mathbf{S}$r o gener$\mathbf{S}$tor on Denote $\mathbf{L}\cdot$ $\mathbf{K}^{-1}\mathbf{L}\mathbf{K}$, i e , $\mathbf{L}$ is the oper$\mathbf{S}$tor $\mathbf{S}$cting on f nctions on $_0$ hich s$\mathbf{S}$tis es $\mathbf{K}\mathbf{L}\mathbf{G}$ $\mathbf{L}\mathbf{K}\mathbf{G}$ Denote by $\mathbf{L}$

Proof A straightforward calculation see shows that

$$L_\varepsilon^{-} k \ \eta \quad - \sum_{x \in \eta} \int_{\mathbb{R}^d} dy\, a_\varepsilon\ x - y\ r\ x, y, \eta \setminus x$$

$$\times \int_{\Gamma_0} \lambda\ d\xi\ \ k\ \xi \cup \eta\ e_\lambda\ e^{\varphi^- \ x - \cdot) - \varphi^+ \ y - \cdot)} - \ , \xi\ ,$$

$$L_\varepsilon^{+} k \ \eta \quad \sum_{y \in \eta} \int_{\mathbb{R}^d} dx\, a_\varepsilon\ x - y\ r\ x, y, \eta \setminus y$$

$$\times \int_{\Gamma_0} \lambda\ d\xi\ \ k\ \xi \cup \ \eta \setminus y\ \cup x\ e_\lambda\ e^{\varphi^- \ x - \cdot) - \varphi^+ \ y - \cdot)} - \ , \xi\ ,$$

$$L_0^{-} k \ \eta \quad - \sum_{x \in \eta} \mathrm{e}\ \mathrm{p}\ E^{\varphi^-}\ x, \eta \setminus x$$

$$\times \int_{\Gamma_0} \lambda\ d\xi\ \ e_\lambda\ e^{\varphi^- \ x - \cdot)} - \ , \xi\ k\ \eta \cup \xi\ ,$$

$$L_0^{+} k \ \eta \quad \sum_{y \in \eta} \mathrm{e}\ \mathrm{p} - E$$

Fourier transforms so that they are unitary operators in $L^2(\mathbb{R}^d \to \mathbb{C}, dx)$. For any $\gamma \in \Gamma$, consider a dynamics of independent particles which starts at γ and such that each separate particle moves according to the semigroup p_t, i.e., independent random walks in $\mathbb{R}^d$. Then, this process has *cádlág* paths on Γ and

and initial distri tion μ_0 his dyna ics can e constr cted as fo lo s, cf). For each $\gamma \in$, denote y P_γ the a of a process on hich is at γ at ti e zero, and after this, points of γ rando ly die, independently of each other, so that the pro a ility that at ti e $t >$ a particle $x \in \gamma$ is still a i e is eq al to e^{-t} Ne t, let π denote the Poisson point process in $\mathbb{R}^d \times$ $, \infty$ ith the intensity ea s re $k_0^{(1)}\, dx\, dt$ he ea s re π is concentrated on config rations $\hat{\gamma}$ $\{ x_n, t_n \}_{n=1}^{\infty}$ in $\mathbb{R}^d \times$ $, \infty$ s ch that $\{x_n\}_{n=1}^{\infty} \in$, $< t_1 < t < \cdots$, and $t_n \to \infty$ as $n \to \infty$ For any s ch config ration, e denote y $P_{\hat{\gamma}}$ the a of a process on s ch that at ti e t , the config ration is e pty, and then at each ti e t_n, $n \in \mathbb{N}$, a ne particle is orn at x_n, and after ti e t_n this particle rando ly dies, independently of the other particles, so that at ti e $s > t_n$ the pro a ility that the particle is still a i e is $e^{-(s-t_n)}$ Finally, the a of the process ith generator and initial distri tion μ_0 is gi en y

$$\int \mu_0(d\gamma)\, P_\gamma * \int \pi(d\hat{\gamma})\, P_{\hat{\gamma}}.$$

ere $*$ stays for con ol tion of ea s res, see) for details

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group ( ) scaled by ε. Set

$$g^\varepsilon(x) := e^{f_0(x)} \int_{\mathbb{R}^d} p^\varepsilon_{t_1}(x, dx_1) \int_{\mathbb{R}^d} p^\varepsilon_{t_2-t_1}(x_1, dx_2) \times \cdots \times \int_{\mathbb{R}^d} p^\varepsilon_{t_n-t_{n-1}}(x_{n-1}, dx_n) \prod_{i=1}^{n} e^{f_i(x_i)}, \quad x \in \mathbb{R}^d.$$

Then, by ( ) and the construction of the process, the first integral in ( ) with $\varepsilon > 0$ is equal to

$$\int_\Theta \prod_{x\in\gamma} g^\varepsilon(x)\, \mu_0(d\gamma) = \sum_{n=1}^{\infty} \frac{1}{n!} \int_{(\mathbb{R}^d)^n} \prod_{i=1}^{n} (g^\varepsilon(x_i) - 1)\, k_0^{(n)}(x_1, \dots, x_n)\, dx_1 \cdots dx_n.$$

In the above integrals, one represents the correlation functions through the Ursell functions, makes a change of variables under the sign of integral, and after a careful analysis of the obtained expression, one takes its limit as $\varepsilon \to 0$. Finally, one shows that the obtained limit is indeed equal to the second integral in ( ).

Convergence of equilibrium Kawasaki dynamics of interacting particles

In this section, we will consider equilibrium dynamics of interacting particles having a Gibbs measure as an equilibrium measure. Our result will extend that of [ ], where just one special case of such a dynamics was considered (see also [ ]). We start with a description of the class of Gibbs measures we are going to use.

A pair potential is a Borel measurable function $\varphi : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ such that $\varphi(-x) = \varphi(x) \in \mathbb{R}$ for all $x \in \mathbb{R}^d \setminus \{0\}$. For $\gamma \in \Gamma$ and $x \in \mathbb{R}^d \setminus \gamma$, we define a relative energy of interaction between a particle at x and the configuration γ as $E(x, \gamma) := \sum_{y\in\gamma} \varphi(x-y)$, provided that the latter sum converges absolutely, and otherwise it is set to be $+\infty$. A grand canonical Gibbs measure corresponding to the pair potential φ and activity $z > 0$

n partic ar, e then ha e $\varphi\ x \ \geq -\ B$, $x \in \mathbb{R}^d$ Ne t, e say that the condition of o acti ity high te perat re regi e is f ed if

$$\int_{\mathbb{R}^d} |e^{-\varphi\ x)} - \ |z\, dx < \ e^{1+\ B\ -1},$$

here B is as in A c assica res t of R e e , says that, nder the ass ption of sta i ity and o acti ity high te perat re regi e, there e ists a Gi s eas re μ corresponding to φ and z, and this eas re has corre ation f nctiona hich satis es conditions i iii of heore , ith s in condition i hich is then ca ed the R e e o nd F rther ore, the corresponding rse f nctions satisfy $u^{\ n)}\ \ ,\cdot,\ldots,\cdot\ \in L^1\ \ \mathbb{R}^{d\ n-1}, dx_1 \cdots dx_n$ for each $n \geq$

n hat fo o s, e i ass e that the potentia φ is a so o nded fro a o e o tside so e nite a in $\mathbb{R}^d$ hich is a ays tr e for any rea istic potentia , since it sho d con erge to zero at in nity paths whose generator is $(L_K, D(L_K))$. If we consider this process with initial distribution μ, then it is an equilibrium process, i.e., it has distribution $\mu_t = \mu$ at any moment of time $t \geq$. Thus, for each $t \geq$, $\mu_t = \mu$ has correlation function which satisfies conditions (i)–(iii) of Theorem

respect to μ, as well as integrals over $\mathbb{R}^d$ with respect to Lebesgue measure. As a result one gets rid of all s[illegible]tions $\square_{x\square y}$. Then, one makes a change of

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