const nt ort ity e note th t independent y on the ue of the ort ity m the considered cont ct ode e hi its ery strong c ustering th t is re ected in the ound on the corre tion functions t ny o ent of ti e t Note th t this e ect on the e e of the co puter si u tion s disco ered re dy in nd no it h s the rigorous the tic for u tion nd c ri c tion A direct consequence of the co petition in the ode is the suppression of such c ustering N e y ssu ing the strong enough co petition nd the ig intrinsic ort ity m e pro e the su Poissoni n ound for the so ution to the o ent equ tions pro ided such ound s true for the initi st te Moreo er e c rify speci c in uences of the const nt nd the density dependent ort ity intensities sep r te y More precise y the ig enough intrinsic ort ity m gi es unifor in ti e ound for e ch corre tion function nd the strong co petition resu ts ensure the regu r sp ti distri ution of the typic con gur tion for ny o ent of ti e th t is re ected in the su Poissoni n ound oint in uence of the intrinsic ort ity nd the co petition e ds to the e istence of the unique in ri nt e sure for our ode hich is just Dir c e sure concen tr ted on the e pty con gur tion The tter e ns th t the corresponding stoch stic e o ution of the popu tion is sy ptotic y e h usting

The e sure μ , $K^{\square}$ is c ed the corre tion e sure of
As sho n in for $\square$ $\mathbf{M}^{1}_{\mathrm{fm}}$ nd ny $G \square L^{1}{}_{0}$ μ the series is , s so ute y con ergent Further ore $KG \square L_{(4G}$

here

$$D_x^+ F \quad F \quad x - F$$

nd $\varkappa^+$ is so e positi e const nt

The e istence of the process ssoci ted ith L_+ c n e sho n using the s e technique s in Let $_t$ e the corresponding e o ution of e sures in ti e on $\mathbf{M}^1_{\mathrm{fm}}$ By $k_t^{(n)}$ $n \geq$ e denote the dyn ics of the corresponding n th order corre tion functions pro ided they e ist Note th t e ch of such functions descri es the density of the syste t the o ent t

Then using for ny continuous on $\mathbb{R}^d$ ith ounded support e o t in

$$\frac{d}{dt}\int_{\mathbb{R}^d} x \, k_t^{(1)} \, x \, dx \quad \frac{d}{dt}\int_\Gamma \quad d \, _t \quad \int_\Gamma L_+ \quad d \, _t$$
$$\varkappa^+ \int_\Gamma a^+ \quad d \, _t \quad \varkappa^+ \int_{\mathbb{R}^d} a^+ \quad x \, k_t^{(1)} \, x \, dx$$
$$\varkappa^+ \int_{\mathbb{R}^d} x \quad a^+ \, k_t^{(1)} \quad x \, dx$$

here denotes the c ssic con o ution on $\mathbb{R}^d$ Hence $k_t^{(1)}$ gro s e ponenti y in t In p rticu r for the tr ns tion in ri nt c se one h s $k_0^{(1)}$ $x \equiv k_0^{(1)}$ nd s resu t

$$k_t^{(1)} \quad e^{\varkappa^+ t} k_0^{(1)}$$

One of the possi i ities to pre ent the density gro th of the syste is to inc ude the de th ech nis The si p est one is descri ed y the independent de th r te ort ity m This e ns th t ny e e ent of popu tion h s n independent e ponenti y distri uted ith p r eter m r ndo ife ti e The independent de th together ith the independent cre tion of ne p rtic es y re dy e isting ones de scri e the so c ed *contact model* in the continuu see e g The pre gener tor of such ode is gi en y the fo o ing e pression

$$L_{\mathrm{CM}} F \quad m \sum_{x \in \gamma} D_x^- F \quad L_+ F$$
$$m \sum_{x \in \gamma} D_x^- F \quad \varkappa^+ \sum_{y \in \gamma} \int_{\mathbb{R}^d} a^+ \, x - y \, D_x^+ F \quad dx$$

here

$$D_x^- F \quad F \quad \setminus x - F$$

The M r o process ssoci ted ith the gener tor L_{CM} s constructed in This construction s gener ized in for ore gener c sses of functions a^+ Let us note th t the cont ct ode in the continuu y e used in the epide io ogy to ode the infection spre ding process The ues of this process represent the st tes of the infected popu tion This is n og of the cont ct process on ttice Of course such interpret tion is not in the sp ti eco ogy concept On the other h nd cont ct process is sp ti r nching process ith gi en ort ity r te

The dyn ics of corre tion functions in the cont ct ode s considered in N e y t ing m for correctness e h e for ny $n \geq$ t the corre tion function of n-th order has the following form

$$k_t^{(n)}(x_1,\dots,x_n) = e^{n(\varkappa^+-1)t}\left[\bigotimes_{i=1}^{n} e^{tL^i_{a^+}}\right] k_0^{(n)}(x_1,\dots,x_n) + \varkappa^+\int_0^t e^{n(\varkappa^+-1)(t-s)}\left[\bigotimes_{i=1}^{n} e^{(t-s)L^i_{a^+}}\right] \times \sum_{i=1}^{n} k_s^{(n-1)}(x_1,\dots,\check{x}_i,\dots,x_n)\sum_{j:\, j\neq i} a^+(x_i-x_j)\,ds$$

where

$$L^i_{a^+}k^{(n)}(x_1,\dots,x_n) = \varkappa^+\int_{\mathbb{R}^d} a^+(x_i-y)\left[k^{(n)}(x_1,\dots,x_{i-1},y,x_{i+1},\dots,x_n) - k^{(n)}(x_1,\dots,x_n)\right]dy$$

and the symbol $\check{x}_i$ means that the i-th coordinate is omitted. Note that $L^i_{a^+}$ is Markov generator and the corresponding semigroup in L^∞ space preserves e consider $t \geq$ One c n pro e y induction th t for ny $\{x_1, \ldots, x_n\} \subset B$ $n \geq$

$$k_t^{(n)}\ x_1, \ldots, x_n \ \geq \ {}^n e^{n(\varkappa^+ - 1)t} n!$$

Indeed for n this st te ent h s een pro ed Suppose th t ho ds for $n -$
Then y one h s

$$\begin{aligned} k_t^{(n)}\ x_1, \ldots, x_n \ &\geq \varkappa^+ \int_0^t e^{n(\varkappa^+ - 1)(t-s)} n \ {}^{(n-1)} e^{(n-1)(\varkappa^+ - 1)s}\ n - \ ! \ n - \ ds \\ &\geq \ {}^n n! e^{n(\varkappa^+ - 1)t} \int_0^t e^{-(\varkappa^+ - 1)s} ds \geq \ {}^n e^{n(\varkappa^+ - 1)t} n! \end{aligned}$$

As it s entioned efore the ter ound sho s the c ustering in the cont ct ode A pre ious consider tion y e e tended for the c se $m \in$, e shou d on y rep ce y m in the pre ious c cu tions

As conc usion e h e the presence of ort ity m $\varkappa^+$ in the free gro th ode pre ents the gro th of density i e the corre tion functions of orders dec y in ti e But it doesn t in uence on the c ustering in the syste One of the possi i ities to pre ent such c ustering is to consider the so c ed density dependent de th r te N e y et us consider the fo o ing pre gener tor

$$LF \qquad \sum_{x \in \gamma}{}^{2}$$

Proof. It is not di cu t to sho th t L_0 is dense y de ned nd c osed oper tor in $\mathbf{L}_C$.

Let $\frac{\pi}{2}$ e r itr ry nd ed C e r th t for ζ Sect $\frac{\pi}{2}$

$$m| \ | \ \varkappa^{-} E^{a^{-}} \qquad \zeta \qquad 0$$

Therefore for ny ζ Sect $\frac{\pi}{2}$ the in erse oper tor $L_0 - \zeta$ $^{-1}$ the ction of hich is gi en y

$$L_0 - \zeta \quad ^{-1} G \qquad - \frac{}{m| \ | \quad \varkappa^{-} E^{a^{-}} \qquad \zeta} G$$

is e de ned on the ho e sp ce $\mathbf{L}_C$ Moreo er it is ounded oper tor in this sp ce nd

$$\| L_0 - \zeta \quad ^{-1} \| \leq \begin{cases} \frac{1}{|\zeta|} & \text{if } \mathrm{Re}\,\zeta \geq \\ \frac{M}{|\zeta|} & \text{if } \mathrm{Re}\,\zeta \end{cases}$$

here the const nt M does not depend on ζ

The c se $\mathrm{Re}\,\zeta \geq$ is direct consequence of nd inequ ity

$$m| \ | \quad \varkappa^{-} E^{a^{-}} \qquad \mathrm{Re}\,\zeta \geq \mathrm{Re}\ \zeta \geq$$

e pro e no the ound in the c se $\mathrm{Re}\,\zeta$ sing e h e

$$\| L_0 - \zeta \quad ^{-1} G \|_C \quad \left\| \frac{}{m| \cdot | \quad \varkappa^{-} E^{a^{-}} \cdot \quad \zeta} G \cdot \right\|_C$$

$$\frac{}{|\zeta|} \left\| \frac{|\zeta|}{m| \cdot | \quad \varkappa^{-} E^{a^{-}} \cdot \quad \zeta} G \cdot \right\|_C$$

Since ζ Sect $\frac{\pi}{2}$

$$|\mathrm{I} \quad \zeta| \geq |\zeta| \ \sin \quad - \qquad |\zeta| \cos$$

Hence

$$\frac{|\zeta|}{m| \ | \quad \varkappa^{-} E^{a^{-}} \qquad \zeta} \leq \frac{|\zeta|}{|\mathrm{I} \quad \zeta|} \leq \frac{}{\cos} \qquad M$$

nd is fu ed

The rest of the st te ent of the e fo o s direct y fro the theore of Hi e Yosid see e g ☐

e de ne no

$$L_1 G \qquad \varkappa^{-} \sum_{x \in \eta} \sum_{y \in \eta \setminus x} a^{-} \ x - y \ G \ \setminus y \qquad G \in D \ L_1 \qquad D \ L_0$$

The e e o i p ies th t the oper tor L_1 is e de ned

Lemma *For any the*

Proof. By odu us property

$$\|L_1G\|_C \quad \int_{\Gamma_0} |L_1G\ \ |C^{|\eta|}\ d$$

c n e esti ted y

$$\varkappa^- \int_{\Gamma_0} \sum_{x\in\eta} \sum_{y\in\eta\setminus x} a^-\ x - y\ |G\ \ \setminus y\ |C^{|\eta|}\ d$$

By Min os e is equ to

$$\varkappa^- \int_{\Gamma_0} \int_{\mathbb{R}^d} \sum_{x\in\eta} a^-\ x - y\ |G\ \ |C^{|\eta|+1} dy\ d$$

$$\varkappa^- \int_{\Gamma_0} C|\ \|G\ \ |C^{|\eta|}\ d \leq \frac{\varkappa^-}{m} C\|L_0G\|_C$$

Therefore ho ds ith

$$a \quad \frac{\varkappa^- C}{m}$$

C e r th t t ing

$$C_0 \quad \frac{m}{\varkappa^-}$$

e o t in th t a for C C_0 □

e set no

$$L_2G\ \ \quad L_{2,\varkappa^+}G\ \ \quad \varkappa^+ \int_{\mathbb{R}^d} \sum_{y\in\eta} a^+\ x - y\ G\ \ \setminus y\ \ x\ dx$$

$$G\ \ D\ L_2\ \quad D\ L_0$$

The oper tor de ned s

$$N G \quad | \, | G \qquad G \, D \, L_0$$

is c ed the nu er oper tor

Remark *We proved, in particular, that for* $G \, D \, L_0 \quad D \, L_1 \quad D \, L_2$

$$\|L_1 G\|_C \leq \varkappa^- C \|NG\|_C$$
$$\|L_2 G\|_C \leq \varkappa^+ \|NG\|_C.$$

Fin y e consider the st p rt of the oper tor L

$$L_3 G \quad \varkappa^+ \int_{\mathbb{R}^d} \sum_{y \, \eta} a^+ \; x - y \; G \quad x \, dx \qquad D \, L_3 \quad D \, L_0.$$

Lemma *For any and any* $\varkappa^+$ *,* C *such that*

$$\varkappa^+ E^{a^+} \qquad C \;\; \varkappa^- E^{a^-} \qquad m| \, |$$

the following estimate holds

$$\|L_3 G\|_C \leq a \|L_0 G\|_C \qquad G \, D \, L_3$$

with $a \quad a \; \varkappa^+ \; C$.

Proof. sing the s e tric s s in the t o pre ious e s e h e

$$\|L_3 G\|_C \qquad \int_{\Gamma_0} |L_3 G \quad | C^{|\eta|} \quad d$$
$$\leq \varkappa^+ \int_{\Gamma_0} \int_{\mathbb{R}^d} \sum_{y \, \eta} a^+ \; x - y \; |G \quad x \, | C^{|\eta|} dx \quad d \, .$$

By Min os e is equ to

$$\frac{\varkappa^+}{C} \int_{\Gamma_0} E^{a^+} \quad |G \quad | C^{|\eta|} \quad d \, .$$

The ssertion of the e is no tri i $\square$

Theorem *Assume that the functions* $a^- \;\; a^+$ *and the constants* $\varkappa^- \;\; \varkappa^+$ *,* m *and* C *satisfy*

$$C \varkappa^- a^- \geq \varkappa^+ a^+,$$
$$m \qquad \varkappa^- C \quad \varkappa^+ .$$

Then, the operator L *is a generator of a holomorphic semigroup* $U_t \;\; t \geq$ *in* $\mathbf{L}_C$.

Proof. The st te ent of the theore fo o s direct y fro Re r Le nd the theore out the pertur tion of ho o orphic se igroup see e g For the re der s con enience e o e gi e its for u tion

For any $T \, \mathbf{H}$ *and for any* *there exists positive constants* , *such that if the operator* A *satisfies*

$$\|Au\| \leq a \|Tu\| \quad b \|u\| \qquad u \, D \, T \quad D \, A$$

with a *,* b *, then* T

Let us denote for $\imath \ \square\ \square$

$$
\begin{aligned}
Sk \ \imath \quad & - \frac{\varkappa^-}{m | \ | \quad \varkappa^- E^{a^-} \ \imath} \sum_{x \square \eta} \int_{\mathbb{R}^d} k \ \ y \ \square \imath \quad a^- \ \ x - y \ \ dy \\
& \frac{\varkappa^+}{m | \ | \quad \varkappa^- E^{a^-} \ \imath} \sum_{x \square \eta} \sum_{y \square \eta \setminus x} a^+ \ x - y \ k \ \imath \ \ \setminus x \\
& \frac{\varkappa^+}{m | \ | \quad \varkappa^- E^{a^-} \ \imath} \int_{\mathbb{R}^d} \sum_{y \square \eta} a^+ \ x - y \ k \ \ \imath \ \ \setminus y \ \ \square x \ dx
\end{aligned}
$$

nd

$$
Sk \quad \square
$$

Let

$$
\square k \square_C \quad \operatorname*{ess\,sup}_{\eta \square \Gamma_0} \frac{| k \ \imath \ \ |}{C^{|\eta|}}
$$

then

$$
\begin{aligned}
& \square Sk \square_C \\
\leq \ & \square k \square_C \operatorname*{ess\,sup}_{\eta \square \Gamma_0 \setminus \{\square\}} \frac{\varkappa^- C}{m | \ | \quad \varkappa^- E^{a^-} \ \imath} \sum_{x \square \eta} \int_{\mathbb{R}^d} a^- \ \ x - y \ \ dy \\
& \frac{\square k \square_C}{C} \operatorname*{ess\,sup}_{\eta \square \Gamma_0 \setminus \{\square\}} \frac{\varkappa^+}{m | \ | \quad \varkappa^- E^{a^-} \ \imath} \sum_{x \square \eta} \sum_{y \square \eta \setminus x} a^+ \ \ x - y \\
& \square k \square_C \operatorname*{ess\,sup}_{\eta \square \Gamma_0 \setminus \{\square\}} \frac{\varkappa^+}{m | \ | \quad \varkappa^- E^{a^-} \ \imath} \int_{\mathbb{R}^d} \sum_{y \square \eta} a^+ \ \ x - y \ \ dx \\
\leq \ & \square k \square_C \frac{C \varkappa^-}{m} \quad \square k \square_C \frac{\varkappa^+}{m} \quad \square k \square_C \frac{}{C} \quad \square k \square_C
\end{aligned}
$$

Then

$$\hat{L}G \quad \begin{cases} \varkappa^{+} \int_{\mathbb{R}^d} \sum_{y \in \eta} a^{+}\ x-y\ G^{(n)}\ \ x\ dx & |\ | \quad n- \\ -\varkappa^{-} \sum_{x \in \eta} \sum_{y \in \eta \setminus x} a^{-}\ x-y\ G^{(n)}\ \setminus y & |\ | \quad n \\ L_{diag} G^{(n)} & |\ | \quad n \\ & \text{other ise} \end{cases}$$

Note th t sgn G $\equiv$ if $|\ |\ n$

Therefore for r itr ry $n \geq$

$$\geq I_n \int_{\Gamma_0} \text{sgn}\ G \cdot L - b \quad G \cdot \quad b \quad n)$$

Since $E^{a^-}(\eta)\geq 0$, for $|\xi|\leq 1$ we get

$$
\begin{aligned}
&\geq \sum_{n=1}^{\infty} I_n \geq -m\sum_{n=1}^{\infty} n\frac{t^nC^n}{n!}|\xi|^n-\varkappa^-\sum_{n=1}^{\infty}\frac{t^nC^n}{n!}\int_{\Lambda^n}E^{a^-}\left(x^{(n)}\right)dx^{(n)}\\
&\quad-\varkappa^+\sum_{n=1}^{\infty}\frac{t^nC^n}{n!}n|\xi|^{n-1}\int_{\Lambda}\int_{\Lambda}a^+(x-y)\,dxdy-b\sum_{n=1}^{\infty}\frac{t^nC^n}{n!}|\xi|^n\\
&\geq -mtC|\xi|e^{Ct|\Lambda|}-\varkappa^-\int_{\Gamma_\Lambda}E^{a^-}(\eta)\,d\lambda_{Ct}(\eta)-\varkappa^+Cte^{Ct|\Lambda|}\int_{\Lambda}\int_{\Lambda}a^+(x-y)\,dxdy\\
&\quad-b\left[e^{Ct|\Lambda|}-1\right]\\
&\geq -mtC|\xi|e^{Ct|\Lambda|}-\varkappa^-C^2t^2\int_{\Gamma_\Lambda}\int_{\Lambda}\int_{\Lambda}a^-(x-y)\,dxdy\,d\lambda_{Ct}(\eta)\\
&\quad-\varkappa^+Cte^{Ct|\Lambda|}\int_{\Lambda}\int_{\Lambda}a^+(x-y)\,dxdy-b\left[e^{Ct|\Lambda|}-1\right]>-\infty.
\end{aligned}
$$

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